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2-GHz watt-level Kerr-lens mode-locked Yb:KGW laser

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Abstract: We report on a 2-GHz high-power Kerr-lens mode-locked Yb:KGW laser pumped by a single-mode fiber laser. The output performance for two different output coupling rates was investigated. Stable bidirectional mode-locking operation at the repetition rate of 2.157 GHz was obtained with a 0.6% output coupler. The average output powers of bidirectional operation are 741 mW and 746 mW, with 123-fs and 126-fs pulse durations, respectively. By using a 1.6% output coupler, unidirectional mode-locking is achieved with 145-fs pulse duration and 1.7-W average output power, which, to the best of our knowledge, is the highest average power from Kerr-lens mode-locked GHz femtosecond oscillators.

1. Introduction

The gigahertz (GHz) repetition rate femtosecond mode-locked lasers are attractive sources in nonlinear bio-imaging and spectroscopy [1], ultra-high-speed optical communication [2,3] and optical frequency comb generation [4]. Optical frequency combs based on GHz femtosecond lasers have larger comb spacing and, therefore, the increased power per comb line improves the signal-to-noise ratio (SNR) in heterodyne beat measurement. Another advantage is that the larger comb spacing make it easier to access individual comb lines. However, for optical frequency comb generation, the two degrees of freedom of repetition rate and carrier envelope offset (CEO) frequency must be detected and stabilized. Whereas the detection and stabilization of repetition rate is straightforward, detecting the CEO frequency is challenging. The most widely used method is to generate coherent octave-spanning supercontinuum in photonic crystal fibers (PCF) and detect the CEO frequency with the standard f-2f interferometer scheme, however, it requires watt-level average power for GHz repetition rate.

As shown in Fig. 1, direct generation of GHz femtosecond pulses with watt-level average power is usually based on Kerr-lens mode-locked (KLM) Ti:Sapphire oscillators [5–9] and passively mode-locked Yb-doped all-solid-state lasers (ASSLs) with semiconductor saturable absorber mirror (SESAM). With the help of high-power green laser pump sources, 10-GHz Ti:sapphire oscillator with the average power of 1.2 W has been demonstrated [5]. Based on a micro-structured fiber, the spectrum of the 10-GHz Ti:sapphire oscillator was broadened to cover from 470 nm to 1130 nm for detecting the CEO frequency beat note in a f-2f interferometer. Combining customized SESAM and suitable cavity design, the SESAM mode-locked GHz Yb-doped ASSLs pumped by high-power multimode laser diodes (LD) can also support watt-level average power. Thanks to the high average power, it is feasible to generate an octave-spanning supercontinuum generation in PCF directly. For example, a 1.06-GHz SESAM mode-locked Yb:KGW laser with 3.4-W average power and 125-fs pulse duration was reported, which drives an octave-spanning frequency comb generation with a 1-m long highly nonlinear PCF directly [10].

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Since then, a series of SESAM mode-locked GHz femtosecond lasers were reported [11–16]. Recently, 172-fs pulses with 1.44-W average power were obtained from a 10-GHz straight-cavity Yb:CALGO laser, in which a dispersion-engineered Si_3N_4 ridge waveguide was used generating an octave-spanning supercontinuum [17].

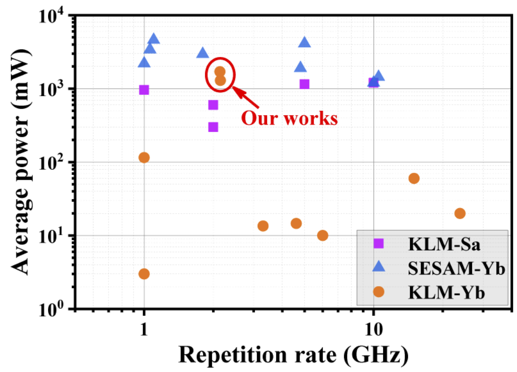

Fig. 1. State of the art of GHz femtosecond oscillators. KLM-Sa: GHz KLM Ti:Sapphire oscillators [5–9]; SESAM-Yb: GHz SESAM mode-locked Yb-based ASSLs [10–17]; KLM-Yb: GHz KLM Yb-based ASSLs [18–24].

Besides GHz KLM Ti:sapphire lasers and SESAM mode-locked Yb-based ASSLs, several GHz-KLM Yb-based ASSLs were also demonstrated [18–23]. As shown in Fig. 1, the highest repetition rate was 23.8 GHz by using a three-element cavity, and the corresponding pulse duration was 140 fs [24]. However, the average power was limited to few tens milliwatts range due to following reasons. The short concave mirror radius in GHz cavities requires a smaller pump beam spot than a circulating laser at the gain crystal to satisfy soft-aperture KLM [25]. This in turn requires an excellent pump beam quality. Therefore, single-mode fiber coupled LDs with maximum power of ∼1 W are used commonly as the pump source. Meanwhile, the output couplers (OC) with transmittance less than 0.4% were usually employed to reduce the cavity loss for maintaining sufficient intracavity power. For frequency comb applications, the low-power GHz pulses require additional amplification, which increases the overall complexity and leads to higher noise. Therefore, it is essential to improve the average power of GHz KLM Yb-based ASSLs.

In this paper, we demonstrated a single-mode fiber laser-pumped high-power KLM Yb:KGW lasers at 2-GHz repetition rate with different OCs. The bidirectional mode locking was obtained with 0.6% OC at the repetition rate of 2.157 GHz. The average power of the two outputs were 741 mW and 746.2 mW, with 123-fs and 126-fs pulse duration, respectively. Meanwhile, unidirectional mode locking was also achieved with 0.6% OC, and the corresponding average output power and pulse duration were 1.29 W and 129 fs. Without active control, the mode locking was so sensitive to the environment that it could only last for about ten minutes. Furthermore, the higher average output power up to 1.7 W was demonstrated by 1.6% OC with 145-fs pulse duration at the repetition rate of 2.141 GHz.

2. Experimental setup

A schematic diagram of the experimental setup is shown in Fig. 2. To induce a strong Kerr-lens effect, a Yb(5 at.%):KGW crystal was used as a gain medium with a high nonlinear coefficient (2 $\times 10^{-15}$ cm^2/W) [26]. The crystal was Ng-cut and had a dimension of 2 mm in length and 3 mm $\times$ 3 mm in cross section. The sample was antireflection coated around 980-1100 nm on both surfaces, and was wrapped with indium foil and tightly mounted in a water-cooled copper heat

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sink to maintain a 13 °C temperature during the experiment. The pump source was a commercial single-mode fiber laser emitting at 976 nm with maximum output power of 10 W (ALS-IR-75, Azur Light Systems). A half-wave plate (HWP) was used to adjust the polarization of the pump beam for higher absorption of Yb:KGW. The collimated pump spot before L1 was 971 µm in diameter, and it was re-collimated to 2.6-mm by using a collimating and defocusing system consisting of a concave lens (L1) with a focal length of 25 mm and a convex lens (L2) with 75-mm focal length. The pump beam was then focused into the crystal via a convex lens (L3) with 40-mm focal length. The focused pump spot diameter was measured to be about 27 µm ($1/e^2$). A standard bow-tie ring cavity consisted of two concave mirrors and two plane mirrors. The cavity length was designed about 140 mm, corresponding to 2.15 GHz repetition rate. Based on the ABCD matrix approach, the beam diameter of the circulating laser in the crystal was calculated to be ~46 µm in continuous-wave (CW) operation, which was suitable for soft-aperture KLM. Both concave mirrors had the same radius of curvature (ROC) of 30 mm, and M1 was a dichroic mirror that is anti-reflection coated for the pump wavelength (808-980 nm) and high-reflection coated for the laser wavelength (1020-1200 nm). M2 was a Gires-Tournois interferometer (GTI) mirror, which can offer -800-fs^2 group delay dispersion (GDD) around 1035-1055 nm. M3 was also a GTI mirror providing a GDD of -500 fs^2 around 1010-1060 nm. In order to ensure the sufficient Kerr-lens effect while obtaining high-power output, the output couplers with different transmittance of 0.6% and 1.6% were used.

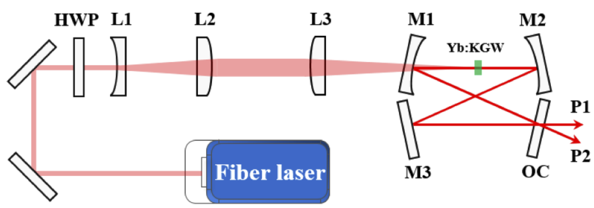

Fig. 2. The layout of the 2-GHz high-power KLM Yb:KGW laser. HWP: half-wave plate; L1: concave lens, f = 25 mm; L2: convex lens, f = 75 mm; L3: convex lens, f = 40 mm; M1: dichroic concave mirror, ROC = 30 mm; Yb:KGW: gain medium; M2: -800 fs^2 GTI mirror, ROC = 30 mm; M3: -500 fs^2 plane GTI mirror; OC: output coupler.

3. Results and discussion

At the beginning, the OC with 0.6% transmittance was used. The cavity was optimized in a bidirectional CW operation close to the inner stability edge [27].

In general, one of the clockwise and counter-clockwise oscillations will disappear when mode locked because cavity alignment caused unequal loss or gain between the two directions [28,29]. However, it is possible to achieve stable bidirectional mode-locking operation if the intra-cavity loss is small, as we obtained with the 0.6% OC. As shown in Fig. 3(a) and Fig. 3 (b), the bidirectional mode-locking output power was 741 mW (P1) and 746 mW (P2) at 10-W pump power, respectively. Operating in open air, the root mean square (RMS) values of power fluctuations of P1 and P2 over one hour were 0.63% and 0.49%. The optical spectrum as well as the temporal properties of the two outputs were measured via a commercial optical spectrum analyzer (AvaSpec-ULS4096CL-EVO) and an intensity auto-correlator (APE PulseCheck USB), respectively. Figure 3(c) and Fig. 3(e) show the mode-locking spectrum and temporal properties of the P1 output. The optical spectrum of the pulses was centered at 1042.4 nm, with the full-width at half-maximum (FWHM) bandwidth of 8.4 nm. The measured intensity auto-correlation trace had an FWHM duration of 190 fs. If assuming a $sech^2$ pulse shape, and the corresponding pulse duration was 123 fs. Figure 3(d) and Fig. 3(f) show the mode-locking spectrum and temporal properties of the P2 output. The FWHM bandwidth of optical spectrum was 8.7 nm

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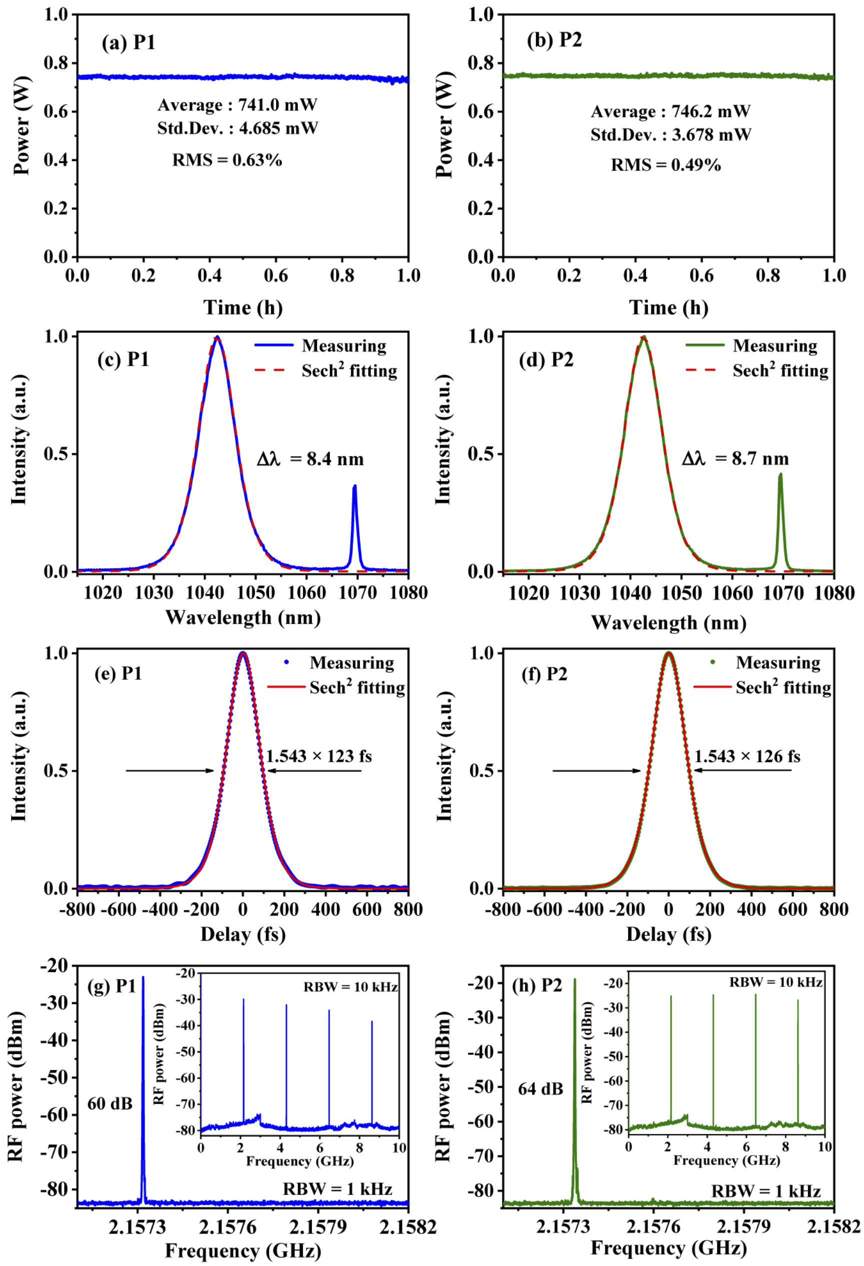

Fig. 3. The characters of bidirectional mode-locking operation with 0.6%OC. (a) (b) The RMS of power fluctuation of P1 and P2 outputs in one hour; (c) (d) The measured mode-locking spectrum of P1 and P2; (e) (f) The corresponding auto-correlation trace of P1 and P2. Blue and green dots are experimental data, pink and red curves are sech^2 fitting; (g) (h)The RF spectrum of P1 and P2 measured with different frequency windows of 2 MHz at 1 kHz of RBW and 10 GHz at 10 kHz of RBW (insets), respectively.

centered at 1042.5 nm. The measured pulse duration was 126 fs assuming a $sech^2$ pulse shape. Furthermore, the radio frequency (RF) spectra of bidirectional mode locking were measured with a photodetector (ET-5000) with bandwidth of 12.5 GHz and a commercial RF spectrum analyzer (Agilent E4407B). As described in Fig. 3(g) and Fig. 3(h), the fundamental frequencies of both output signals were about 2.157 GHz with a resolution bandwidth (RBW) of 1 kHz spanning 2 MHz. The signal-to-noise ratios (SNR) were 60 dB and 64 dB, respectively. The insets show the RF spectra of the pulse trains in a 10-GHz region with RBW of 10 kHz. No obvious side peaks of the several harmonics of the fundamental frequency were observed, which indicates that the bidirectional KLM runs stably.

The mode-locking in a unidirectional or bidirectional operation was stochastically triggered by adjusting the mirror M2 manually in our experiment, and stable bidirectional mode locking was observed without the help of a hard aperture for the negative feedback [30]. For bidirectional mode locking, the repetition rates of the two outputs can be identical or slightly different depending on the cavity alignment. In our case, we did not observe the two different signal of fundamental repetition rate or beating signal by detecting two spatially overlapped lasers with the same polarization. We believe that due to almost the same output power and pulse durations of the counter-propagation pulses, the difference in repetition rate is negligible. Nevertheless, since the direction-dependent self-steepening effect in the gain medium changed the group velocity of the bidirectional pulses according to the optical intensity, bidirectional operation with a slight difference in repetition rate can be achieved by adjusting the cavity alignment [31–33]. In the following work, the 2-GHz mode-locked Yb:KGW laser will be optimized in asymmetrical Kerr effect for bidirectional operation with slightly different repetition rates, making the laser a potential tool for dual-comb spectroscopy.

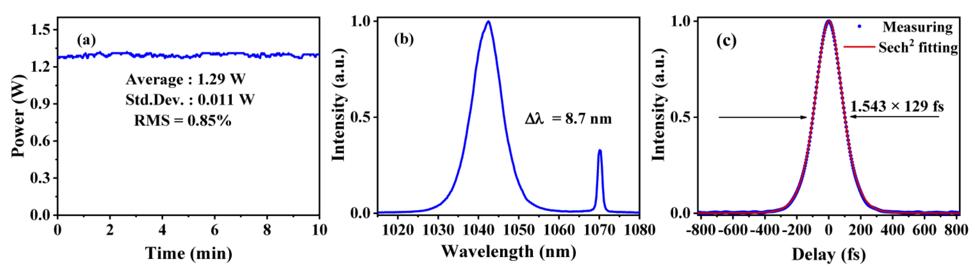

Fig. 4. The characteristics of the unidirectional mode-locking operation with 0.6% OC. (a) The RMS of power fluctuation in ten minutes; (b) The measured mode-locking spectrum; (c) The corresponding auto-correlation trace. Blue dots and pink curve are experimental data and $sech^2$ fitting, respectively.

Apart from the stable bidirectional mode locking with 0.6% OC, the unidirectional operation was also achieved. However, it cannot run stably for longer time compared to the bidirectional mode locking. At the maximum pump power, the average output power was 1.29 W. But after ten minutes of free running, the unidirectional mode locking switched to stable bidirectional operation automatically. The RMS of power fluctuation in ten minutes was 0.85% in open air as shown in Fig. 4(a). The unidirectional mode-locking optical spectrum was centered at 1042.2 nm with FWHM of 8.7 nm. The corresponding pulse duration measured was 129 fs assuming a $sech^2$ pulse shape, as shown in Fig. 4(b) and Fig. 4(c).

For obtaining higher output power, the OC with transmittance of 1.6% was used. After optimizing the CW operation, the output power of P1 and P2 were 1.094 W and 0.893 W. In this case, the stable mode locking was achieved by optimizing the positions of crystal and the mirror M2 slightly. At the maximum pump power, unidirectional mode locking with 1.7-W average output power was obtained. The corresponding optical-to-optical efficiency was 17%, which is

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better than that of the reported multi-GHz KLM Ti:Sapphire oscillators (~10%) and multi-GHz KLM Yb-based ASSLs (less than 6%). As shown in Fig. 5(a), the RMS of the power fluctuation was 0.76% over 40 minutes free running in the open air. Figure 5(b) show the optical spectrum with FWHM of 8.8 nm centered at 1042.2 nm. The measured pulse duration was 145 fs assuming a sech^2 pulse shape, as shown in Fig. 5(c). Compared to the unidirectional operation of 0.6% OC, the increased pulse duration was due to the decreased intracavity power. Figure 5(d) shows a distinct SNR as high as 65 dB of the fundamental beat at 2.141 GHz with a RBW of 1 kHz. The RF spectrum was clean without side peaks or harmonics of the fundamental frequency. The modulation-free higher harmonics in the 10 GHz region with RBW of 10 kHz as shown in the inset confirm clean mode locking.

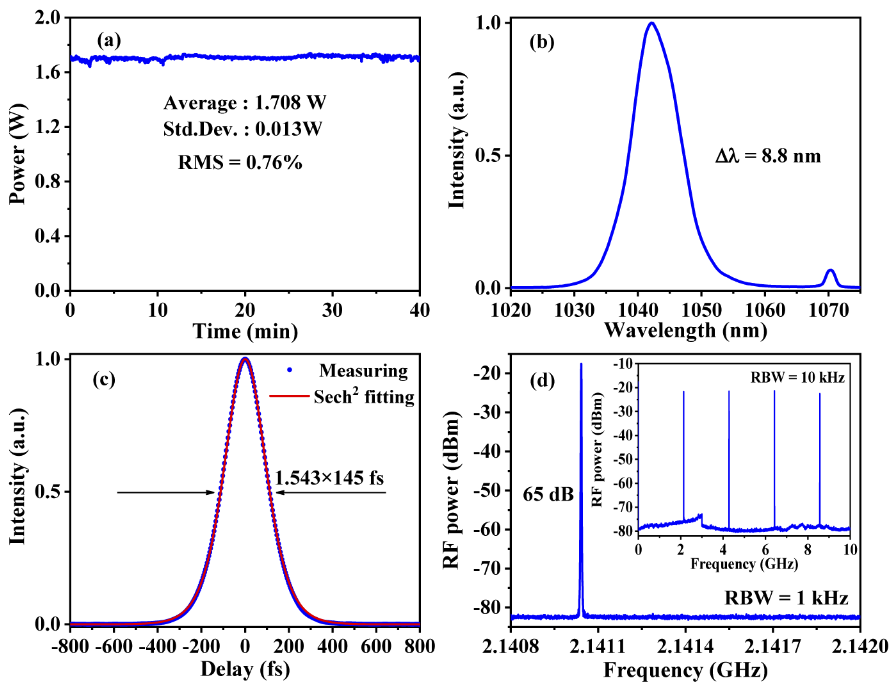

Fig. 5. The characteristics of unidirectional mode-locking operation with 1.6% OC. (a) The RMS power fluctuations in 40 minutes; (b)The measured mode-locking spectrum; (c)The corresponding auto-correlation trace. Blue dots and pink curve are the experimental data and sech^2 fitting, respectively. (d) The RF spectrum measured with different frequency windows of 2 MHz at 1 kHz of RBW and 10 GHz at 10 kHz of RBW (inset), respectively.

It is worth noting that the dispersive wave at 1070 nm was present in all the mode-locked spectra, which is caused by the limited bandwidth of our GTI mirrors (M2: -800 fs^2 at 1035-1055 nm; M3: -500 fs^2 at 1010-1060 nm). Because of the high repetition rate, the limited number of available optical elements prevents the suitable combination of GTI mirrors with different bandwidth for fine dispersion compensation. For obtaining shorter pulse duration, custom GTI mirrors are necessary.

4. Conclusion

In conclusion, the excellent optical properties of the Yb:KGW crystal (e.g., high nonlinear refractive index) allow multi-GHz operation even as the net cavity losses were increased by using OCs with high transmittance. We demonstrated high-power KLM Yb:KGW lasers with

GHz repetition rate. Stable bidirectional and unidirectional operations were achieved with different output couplers of 0.6% and 1.6% transmittance, respectively. Using 0.6% OC, stable bidirectional mode locking with average output power of 741 mW and 746 mW oscillating in two directions was realized, with 123 fs and 126 fs pulse duration of the two outputs, respectively. The mode locking either in a uni- or bi-directional manner was triggered stochastically by adjusting the mirror M2 manually. In the case of unidirectional mode locking, 1.29-W average output power with 129-fs pulse duration was achieved, but the laser operated for a short time. We believe that this unidirectional operation could be more insensitive to the environment if we house it or take some active stabilization. The repetition rate of both mode-locking states was 2.157 GHz. Using 1.6% OC, and unidirectional mode locking with higher average output power up to 1.7 W was obtained. The optical-to-optical efficiency was 17% and the measured pulse duration was 145 fs. The repetition rate was 2.141 GHz, and the corresponding pulse energy and peak power was 0.79 nJ and 5.4 kW, respectively. Higher output power is possible with higher pump power and broadband GTI mirrors for dispersion compensation. For frequency comb applications, the peak power of our oscillator is not enough for spectral broadening in an f-2f interferometer, which limits its applicability as a frequency comb source. However, the watt-level GHz oscillator could be stabilized to optical frequency standards through a cavity stabilized CW laser. Moreover, the bidirectional KLM ring laser will be developed for the emission of two GHz-femtosecond pulse trains with slightly different repetition rates, which is attractive for dual-comb spectroscopy [34].

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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