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Universality of the Dynamic Characteristic Relationship of Electron Correlation in the Two-Photon Double Ionization Process of a Helium-Like System

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Universality of the dynamic characteristic relationship between the characteristic time t_c and the two-electron Coulomb interaction energy $\overline{V}_{12}$ of the ground state in the two-photon double ionization process is investigated via changing the parameters of the two-electron atomic system and the corresponding laser conditions. The numerical results show that the product $t_c\overline{V}_{12}$ keeps constant around 4.1 in the cases of changing the nucleus charge, the electron charge, the electron mass, and changing simultaneously the nucleus charge and the electron charge. These results demonstrate that the dynamic characteristic relationship in the two-photon double ionization process is universal. This work sheds more light on the dynamic characteristic relationship in ultrafast processes and may find its application in measurements of attosecond pulses.

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Electron correlation plays an important role in dynamic processes for a many-body system,[1−11] where the time-dependent characteristic of the electron correlation is hard to be identified because of its attosecond timescale. Until recently, with the development of ultrafast laser technology and attosecond high-harmonic pulses, the investigation of the electron correlation in ultrafast dynamic processes becomes accessible in experiments. For example, the relative photo-emission timing is measured by pump-probe experiments,[6] the electron correlation effects in the ionization processes of D_2 molecules have been uncovered successfully by combining ultrafast and synchrotron XUV sources with electron-ion 3D coincidence imaging techniques.[8] These experimental techniques may be applied to strictly test some fundamental theoretical predictions.

A helium-like system is one of the simplest few-body systems, which provides a benchmark system for the study of the time characteristics of the electron correlation effects in many-body dynamic processes. As mentioned by Hu,[7] the electron correlation for the ground state of helium-like atoms becomes stronger with the increase of the nuclear charge. Therefore, one expects that the dynamic process will also change for differently correlated electron systems. Hence, there appears an interesting question: Is there any general characteristics of the electron correlation which may be hold even if the parameters of the system changes? In this work, we demonstrate that the dynamic characteristic relationship between the electron correlation and the characteristic time of the two-photon double ionization (TPDI) process of a two-electron system is valid for various helium-like systems, e.g., Li^+ and Be^{2+} ions. These results consolidate the universality of the dynamic characteristic relationship obtained in our previous work,[12] which can be regarded as a general dynamic characteristic relationship in the atomic ionization processes.

We investigate the TPDI of the helium-like ion system by numerically solving the time-dependent Schrödinger equation (TDSE). In general, the field-free three-particle Schrödinger equation can be written as

$$\Big(-\frac{\hbar^2}{2M}\nabla_{R_0}^2-\frac{\hbar^2}{2m}\nabla_{R_1}^2-\frac{\hbar^2}{2m}\nabla_{R_2}^2-\frac{Ze^2}{|\boldsymbol{R}_1-\boldsymbol{R}_0|}-\frac{Ze^2}{|\boldsymbol{R}_2-\boldsymbol{R}_0|}+\frac{e^2}{|\boldsymbol{R}_1-\boldsymbol{R}_2|}\Big)\Psi(\boldsymbol{R}_0,\boldsymbol{R}_1,\boldsymbol{R}_2)=E_{\rm NR}\Psi(\boldsymbol{R}_0,\boldsymbol{R}_1,\boldsymbol{R}_2), \quad (1)$$

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where M is the nucleus mass, m is the electron mass, $\hbar$ is the reduced Planck constant, Z is the atomic number, e is the elementary charge, $\boldsymbol{R}_0$ is the position vector of the nucleus, $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the position vectors of the two electrons, respectively, and $E_{\rm NR}$ is the non-relativistic energy. On making the standard transformation to the scaled center-of-mass coordinate system,[13] Eq. (1) is reduced to the dimensionless form

$$H_0\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)=E_n\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2), \tag{2}$$

with

$$H_0=\left[-\frac{1}{2\gamma}\nabla_{r_1}^2-\frac{1}{2\gamma}\nabla_{r_2}^2-\frac{Z\xi}{r_1}-\frac{Z\xi}{r_2}+\frac{\xi^2}{|\boldsymbol{r}_1-\boldsymbol{r}_2|}\right], \tag{3}$$

where γ and ξ are the parameters of the helium-like system, with (1) $\gamma=1$ and $\xi=1$ for the case that only the nucleus charge Ze changes, including $Z=3$ for Li^+ ion and $Z=4$ for Be^{2+} ion; (2) $\gamma=1$ and $\xi=b$ for the case that the electron charge changes from $-e$ into $-be$; (3) $\gamma=n$ and $\xi=1$ for the case that the mass of the electron changes from m into nm. Then the eigenenergy E_n can be expressed as $E_n=E_{\rm NR}/E_{\rm s}$ in the case of ignoring the movement of the center of mass, where, in the dimensionless process, $E_{\rm s}=e^2/a_\mu$ is the unit of energy and $a_\mu=\hbar^2/\mu e^2$ is the unit of length with the reduced mass $\mu=Mm/(M+m)\approx m$. The field-free Schrödinger equation (2) can be solved through the variational method, and $\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)$ can be obtained by selecting an appropriate trial wave function, which can be expressed by B-spline functions as

$$\begin{aligned}\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)=&\sum_{i_\alpha,i_\beta,l_\alpha,l_\beta}C_{i_\alpha,i_\beta,l_\alpha,l_\beta}[1+(-1)^SP_{12}]\\&\cdot B_{i_\alpha}^k(r_1)B_{i_\beta}^k(r_2)\\&\cdot\sum_{m_\alpha m_\beta}\langle l_\alpha m_\alpha l_\beta m_\beta|LM\rangle Y_{l_\alpha}^{m_\alpha}(\hat{\boldsymbol{r}}_1)Y_{l_\beta}^{m_\beta}(\hat{\boldsymbol{r}}_2),\end{aligned} \tag{4}$$

where P_{12} is the permutation operator between electrons 1 and 2, $B_{i_\alpha}^k(r_1)$ and $B_{i_\beta}^k(r_2)$ are two B-spline functions of order k,[14,15] $\langle l_\alpha m_\alpha l_\beta m_\beta|LM\rangle$ is the Clebsch–Gordan coefficient, $Y_{l_\alpha}^{m_\alpha}(\hat{\boldsymbol{r}}_1)$ and $Y_{l_\beta}^{m_\beta}(\hat{\boldsymbol{r}}_2)$ are the two spherical harmonics, and S, L, and M are, respectively, the total spin of the two electrons, the total orbital angular momentum, and its z-component.

Once the field-free H_0 eigenfunction $\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is determined, the interaction between the helium-like system and ultrashort laser pulse can be solved. In the dipole approximation and the gauge transformation,[16] the TDSE reduces to

$$i\hbar\frac{\partial}{\partial t}\Phi(\boldsymbol{r}_1,\boldsymbol{r}_2,t)=[H_0+H_{\rm int}(t)]\Phi(\boldsymbol{r}_1,\boldsymbol{r}_2,t), \tag{5}$$

where

$$H_{\rm int}(t)=\xi e\boldsymbol{E}(t)\cdot(\boldsymbol{r}_1+\boldsymbol{r}_2), \tag{6}$$

and $\boldsymbol{E}(t)$ is the electric field of laser pulse. In our simulation, the vector potential of laser pulse is expressed as

$$\boldsymbol{A}(t)=-A_0e^{-(2\ln 2)(t-t_{\rm c})^2/\tau^2}\sin(\omega t)\hat{\boldsymbol{e}}_z, \tag{7}$$

and the corresponding electric field can be expressed as

$$\begin{aligned}\boldsymbol{E}(t)=&E_0e^{-(2\ln 2)(t-t_{\rm c})^2/\tau^2}\cos(\omega t)\hat{\boldsymbol{e}}_z\\&-(4\ln 2)E_0(t-t_{\rm c})\\&\cdot e^{-(2\ln 2)(t-t_{\rm c})^2/\tau^2}\sin(\omega t)/(\omega\tau^2)\hat{\boldsymbol{e}}_z,\end{aligned} \tag{8}$$

where $E_0=A_0\omega$ is the electric-field amplitude, $t_{\rm c}$ is the position of the laser pulse center on the time-axis, τ is the full width at half maximum (FWHM), ω is the central frequency of the laser pulse, and $\hat{\boldsymbol{e}}_z$ is the unit vector of the polarization direction of the laser pulse. Specially, when the electron charge changes from $-e$ into $-be$, Eq. (6) becomes

$$H_{\rm int}(t)=be\boldsymbol{E}(t)\cdot(\boldsymbol{r}_1+\boldsymbol{r}_2). \tag{9}$$

The time-dependent wave function $\Phi(\boldsymbol{r}_1,\boldsymbol{r}_2,t)$ can be expanded in terms of the field-free H_0 eigenfunctions $\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)$. By substituting the expanded expression of $\Phi(\boldsymbol{r}_1,\boldsymbol{r}_2,t)$ into Eq. (5), one can obtain a set of coupled differential equations, which can be solved by the Adams method.[17] Once the time-dependent wave function $\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,t)$ is determined, the probability distribution at the time $t_{\rm f}$ for the two ionized electrons escaped with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ is obtained according to

$$P(\boldsymbol{k}_1,\boldsymbol{k}_2)=|\langle\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)|\Phi(\boldsymbol{r}_1,\boldsymbol{r}_2,t_{\rm f})\rangle|^2, \tag{10}$$

where $\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is the wave function of the uncorrelated double continuum state, which can be expressed as a symmetrized product of two independent-particle Coulomb wave functions.[18] Therefore, the energy distribution of two ionized electrons can be expressed as

$$P(E_1,E_2)=\iint k_1k_2P(\boldsymbol{k}_1,\boldsymbol{k}_2)d\hat{\boldsymbol{k}}_1d\hat{\boldsymbol{k}}_2, \tag{11}$$

where $E_1=k_1^2/2$ and $E_2=k_2^2/2$ are the energies of the two ionized electrons. Especially, one may know that the differential equations of the Coulomb wave function of the hydrogen-like system, caused by the changing of the nucleus charge, the electron charge or the electron mass, having the same form. However, the field-free two-electron wave function $\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)$ does not have such a property. In other words, the changes of field-free two-electron wave function $\Psi_n(\boldsymbol{r}_1,\boldsymbol{r}_2)$ caused by changing the nucleus charge, the electron charge, or the electron mass are independent of each other.

The universality of the dynamic characteristic relationship in the TPDI process can ensure its broad applicability in various correlated systems. Thus various independent correlated systems can be selected as testing samples of the universality of the dynamic characteristic relationship. The correlated system consisting of a nucleus and two electrons has the inherent properties such as mass and charge. When the mass or charge of a particle is changed, the whole three-body correlated system is changed accordingly, and these changes are independent as mentioned above. Therefore, it is reasonable to explore the universality of the dynamic characteristic relationship in the TPDI process by changing these properties of the correlated systems separately.

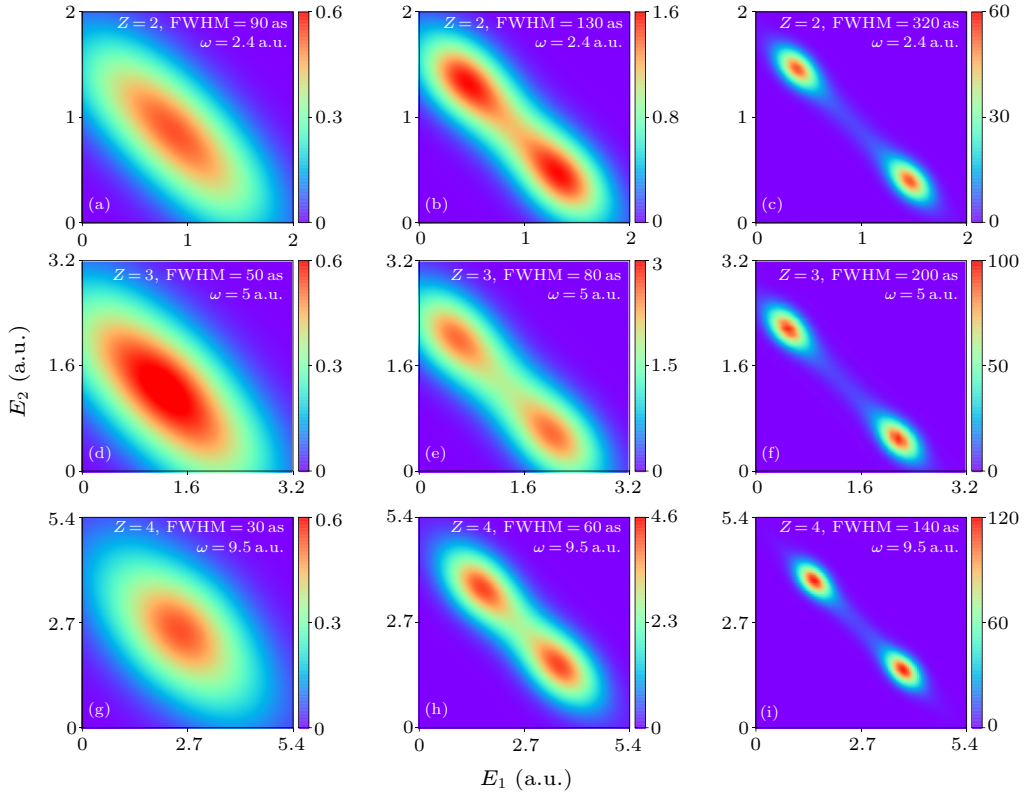

Fig. 1. Energy distribution of two escaped electrons of (a)–(c) helium atom, (d)–(f) Li^+ ion, (g)–(i) Be^{2+} ion exposed to different laser pulse. The laser pulse has a Gaussian envelope around the peak intensity of 1×10^{14} W/cm^2. The center photon energy is 2.4 a.u. for helium atom, 5.0 a.u. for Li^+ ion and 9.5 a.u. for Be^{2+} ion. The FWHM is 90 as, 130 as, 320 as for helium atom, 50 as, 80 as, 200 as for Li^+ ion, and 30 as, 60 as, 140 as for Li^+ ion. The colors bars are in units of 10^{-6} for helium atom, 10^{-10} for Li^+ ion and Be^{2+} ion.

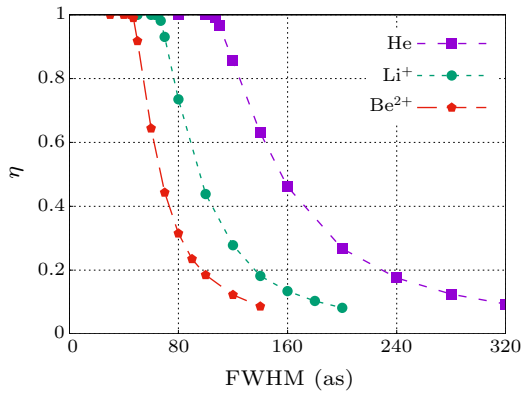

Fig. 2. The ratio η varies with the FWHM of the laser pulse. The laser pulse has a Gaussian envelope around the peak intensity of 1×10^{14} W/cm^2. The center photon energy is 2.4 a.u. for helium atom, 5.0 a.u. for Li^+ ion and 9.5 a.u. for Be^{2+} ion.

We first investigate the TPDI process of helium atom, Li^+ and Be^{2+} ions to further understand the characteristic time. Figure 1 shows the energy distributions of two ionized electrons for helium atom [(a)–(c)], Li^+ [(d)–(f)] and Be^{2+} ions [(g)–(i)], where the energy distribution in the TPDI process changes from one peak into two peaks as the laser pulse duration increases. Hence, we may define a ratio $\eta = P_{\rm mid}/P_{\rm max}$,[12] where $P_{\rm max}$ is the maximum value of the ionization probability in energy space with its coordinates $[E_1(P_{\rm max}), E_2(P_{\rm max})]$, and $P_{\rm mid}$ is the probability at the intersection of lines $E_1 + E_2 = E_{\rm t}$ and $E_1 = E_2$, with E_1 and E_2 being the energies of the two electrons and $E_{\rm t} = E_1(P_{\rm max}) + E_2(P_{\rm max})$. Figure 2 presents the ratio η as a function of pulse duration with the laser intensity of 1×10^{14} W/cm^2 for helium, Li^+ and Be^{2+} ions. Based on our previous work,[12] we define $t_{\rm c}$ as the characteristic time in the two-photon double ionization process, where it is expressed as pulse duration for the turning point of the

energy distribution of two ionized electrons from one peak to two peaks, hence the value of t_c is evaluated based on the pulse duration for the turning point of the ratio η from $\eta = 1$ to $\eta < 1$. More specifically, as shown in Fig. 2, the characteristic time t_c is about 105 as, 65 as and 46 as for helium, Li^+ and Be^{2+} ion, respectively. When the pulse duration is less than t_c, the ratio η equals 1, which indicates that two ionized electrons carry mainly equal energy. In contrast, when the pulse duration is greater than t_c, the ratio η is less than 1 and gradually tends to a constant with the increase of the pulse duration, which means that two ionized electrons carry mainly unequal energy. In order to understand the phenomenon that this ratio tends to a constant, we define the energy difference ΔE for two ionized electrons that possess the maximum ionization probability in the TPDI process. Figure 3 shows the energy difference ΔE as a function of pulse duration with the laser intensity of $1 \times 10^{14}\,\mathrm{W/cm^2}$ for helium, Li^+ and Be^{2+} ions. By comparing Figs. 2 and 3, we may find that, as the pulse duration increases further, the energy difference ΔE tends to be the energy difference of $|I_{p2} - I_{p1}|$ when the ratio η tends to a constant, where I_{p1} and I_{p2} are the first and second ionization energies of the atom or the ions, respectively. It is known that in the sequential TPDI process the energies carried by two ionized electrons are $E_1 = \omega - I_{p1}$ and $E_2 = \omega - I_{p2}$, respectively. Hence the energy difference of two ionized electrons, $\Delta E = E_1 - E_2 = |I_{p2} - I_{p1}|$, can be regarded as a sign of the sequence TPDI in turn based on previous works.[1,3,19] Therefore, the pulse duration for the ratio tending to a constant can be regarded as the intrinsic maximum time delay[3] between the two ionization events, which can lead to a specific combination of final energies of the ejected electrons, i.e., $E_1 = \omega - I_{p1}$ and $E_2 = \omega - I_{p2}$. Moreover, it should be emphasized that the characteristic time we defined here is different from the maximum time delay, which is defined in Ref. [3] and our characteristic time is the minimum duration of the laser pulse where the ejected electrons may carry different energies at the end of the laser pulse, rather than the pulse duration that identifies the nonsequential ionization mechanisms. In addition, we find that the curves of ratio η and energy difference ΔE remain unchanged as the laser intensity decreases from $1 \times 10^{14}\,\mathrm{W/cm^{-2}}$ to $1 \times 10^{13}\,\mathrm{W/cm^{-2}}$ and $1 \times 10^{12}\,\mathrm{W/cm^{-2}}$.

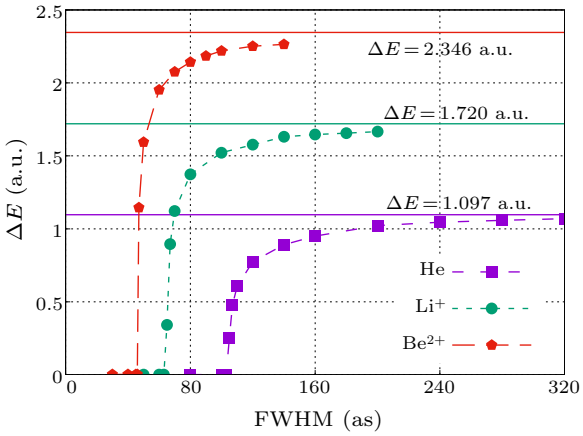

Fig. 3. The energy difference ΔE versus the FWHM of the laser pulse. The laser pulse has a Gaussian envelope around the peak intensity of $1 \times 10^{14}\,\mathrm{W/cm^2}$. The center photon energy is 2.4 a.u. for helium atom, 5.0 a.u. for Li^+ ion and 9.5 a.u. for Be^{2+} ion. The horizontal lines represent the energy difference $\Delta E = |I_{p2} - I_{p1}|$.

Table 1. The parameters for the case of the change of the nucleus charge. The nucleus charge Q_c, the ground state energy E_{1S}, the two-electron Coulomb interaction energy $\overline{V}_{12}$, the second ionization energy I_{p2}, the central frequency ω, and the characteristic time t_c obtained by the time-dependent Schrödinger equation.

Q_c	E_{1S} (a.u.)	$\overline{V}_{12}$ (a.u.)	I_{p2} (a.u.)	ω (a.u.)	t_c (as)	t_c (a.u.)	$\overline{V}_{12} \times t_c$
$1.5e$	−1.465	0.636	1.125	1.3	158	6.529	4.152
$1.732e$	−2.070	0.780	1.5	1.8	127	5.248	4.093
$2e$	−2.903	0.947	2	2.4	105	4.339	4.109
$2.236e$	−3.755	1.094	2.5	3	91	3.760	4.114
$2.45e$	−4.624	1.227	3	3.4	81	3.347	4.107
$2.5e$	−4.811	1.258	3.125	3.6	79	3.264	4.107
$2.8e$	−6.244	1.445	3.92	4.7	70	2.893	4.180
$3e$	−7.28	1.568	4.5	5	65	2.686	4.212
$4e$	−13.654	2.193	8	9.5	46	1.901	4.196

We then investigate the change of the characteristic time by changing the charges of the nucleus and the electron separately. Table 1 presents the corresponding parameters of the helium-like system as the nucleus charge increases from $1.5e$ to $4e$. The ground-state energy E_{1S}, the second ionization energy I_{p2} and the central frequency of the laser pulse are presented in Table 1, where the central frequency of the laser pulse, ω, is chosen to be greater than the second ionization energy and less than the ground-state energy. As mentioned in our previous work,[12] $\overline{V}_{12} = \langle \Psi_{1S}(\boldsymbol{r}_1, \boldsymbol{r}_2) | 1/|\boldsymbol{r}_1 - \boldsymbol{r}_2| | \Psi_{1S}(\boldsymbol{r}_1, \boldsymbol{r}_2) \rangle$ is defined as the two-electron Coulomb interaction energy of the ground state, where $\Psi_{1S}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is the wave function of the ground state. As shown in Table 1, the Coulomb interaction energy of the ground state $\overline{V}_{12}$ increases from 0.636 a.u. to 2.193 a.u., and the corresponding characteristic time t_c of the TPDI process decreases

from 6.529 a.u. to 1.901 a.u., i.e., from 158 as to 46 as. These results can be understood directly as follows: as the charge of the nucleus increases, the two electrons are closer to each other, hence the Coulomb interaction energy increases, and the electron correlation becomes stronger. On the other hand, the characteristic time decreases drastically with the increase of the nucleus charge. The interesting result is that the product of the Coulomb interaction energy of the ground state and the characteristic time of the TPDI process for the systems with different nucleus charges almost keeps constant around 4.1.

The change of the charge and mass of electron to test the universality of the dynamic characteristic relationship can be understood from the perspective of the equivalent quality and charge,[20,21] although they are fundamental physical constants. Table 2 presents the values of the characteristic time and the Coulomb interaction energy of the ground state for the case that the electron charge is changed from $-0.6e$ to $-1.3e$. It shows that the Coulomb interaction energy of the ground state increases from 0.230 a.u. to 1.884 a.u. with the increase of the absolute value of the electron charge, and the corresponding characteristic time of the TPDI process decreases from 18.347 a.u. to 2.190 a.u., i.e., from 444 as to 53 as. However, it is found that the product of the Coulomb interaction energy of the ground state and the characteristic time of the TPDI process also keeps constant around 4.1. Table 3 shows the values of the characteristic time and the Coulomb interaction energy as the nucleus charge increases from $1.4e$ to $2.4e$ and the electron charge changes from $-0.7e$ to $-1.2e$ simultaneously. Table 3 illustrates that although the Coulomb interaction energy of the ground state increases from 0.227 a.u. to 1.964 a.u. and the characteristic time of the TPDI process decreases from 18.058 a.u. to 2.066 a.u., i.e., from 437 as to 50 as, the product of the Coulomb interaction energy of the ground state and the characteristic time of the TPDI process once again almost remains constant around 4.1.

Table 2. The parameters for the case of changing electron charge, i.e., the electron charge Q_e, the ground state energy E_{1S}, the two-electron Coulomb interaction energy $\overline{V}_{12}$, the second ionization energy I_{p2}, the central frequency ω, and the characteristic time t_c obtained by the time-dependent Schrödinger equation.

Q_e	E_{1S} (a.u.)	$\overline{V}_{12}$ (a.u.)	I_{p2} (a.u.)	ω (a.u.)	t_c (as)	t_c (a.u.)	$\overline{V}_{12} \times t_c$
$-0.6e$	−1.190	0.230	0.72	0.8	444	18.347	4.220
$-0.7e$	−1.568	0.355	0.98	1.2	284	11.736	4.166
$-0.8e$	−1.983	0.515	1.28	1.8	196	8.099	4.171
$-0.9e$	−2.429	0.712	1.62	2.1	140	5.785	4.119
e	−2.903	0.947	2	2.4	105	4.339	4.109
$-1.1e$	−3.400	1.221	2.42	3	81	3.347	4.087
$-1.2e$	−3.916	1.533	2.88	3.5	64	2.645	4.054
$-1.3e$	−4.448	1.884	3.38	4	53	2.190	4.126

Table 3. The parameters for the case of changing nucleus charge and electron charge, i.e., the nucleus charge Q_c, the electron charge Q_e, the ground state energy E_{1S}, the two-electron Coulomb interaction energy $\overline{V}_{12}$, the second ionization energy I_{p2}, the central frequency ω, and the characteristic time t_c obtained by the time-dependent Schrödinger equation.

Q_c	Q_e	E_{1S} (a.u.)	$\overline{V}_{12}$ (a.u.)	I_{p2} (a.u.)	ω (a.u.)	t_c (as)	t_c (a.u.)	$\overline{V}_{12} \times t_c$
$1.4e$	$-0.7e$	−0.697	0.227	0.480	0.52	437	18.058	4.099
$1.6e$	$-0.8e$	−1.189	0.388	0.819	1.0	256	10.579	4.104
$1.8e$	$-0.9e$	−1.905	0.621	1.312	1.6	159	6.570	4.080
$2e$	$-e$	−2.903	0.947	2	2.4	105	4.339	4.109
$2.2e$	$-1.1e$	−4.250	1.386	2.928	3.6	71	2.934	4.066
$2.4e$	$-1.2e$	−6.019	1.964	4.147	5	50	2.066	4.058

Table 4. The parameters for the case of changing electron mass, i.e., the electron mass m_e, the ground state energy E_{1S}, the two-electron Coulomb interaction energy $\overline{V}_{12}$, the second ionization energy I_{p2}, the central frequency ω, and the characteristic time t_c obtained by the time-dependent Schrödinger equation.

m_e	E_{1S} (a.u.)	$\overline{V}_{12}$ (a.u.)	I_{p2} (a.u.)	ω (a.u.)	t_c (as)	t_c (a.u.)	$\overline{V}_{12} \times t_c$
$0.4m$	−1.161	0.379	0.8	0.95	262	10.826	4.103
$0.6m$	−1.742	0.568	1.2	1.3	174	7.190	4.084
$0.8m$	−2.322	0.758	1.6	1.7	129	5.331	4.041
m	−2.903	0.947	2	2.4	105	4.339	4.109
$1.2m$	−3.483	1.136	2.4	3	86	3.554	4.037
$1.4m$	−4.064	1.326	2.8	3.4	75	3.099	4.110
$1.6m$	−4.644	1.515	3.2	4.2	65	2.686	4.069

We finally study how the product of the characteristic time of the TPDI process and the Coulomb interaction energy of the ground state changes when the electron mass are changed. Table 4 shows that as the electron mass increases from $0.4m$ to $1.6m$, the Coulomb interaction energy of the ground state increases from 0.379 a.u. to 1.515 a.u., the characteristic time of the TPDI process decreases from 10.826 a.u. to 2.686 a.u., i.e., from 262 as to 65 as, and also the product of the Coulomb interaction energy of the ground state and the characteristic time of the TPDI process almost remains constant around 4.1.

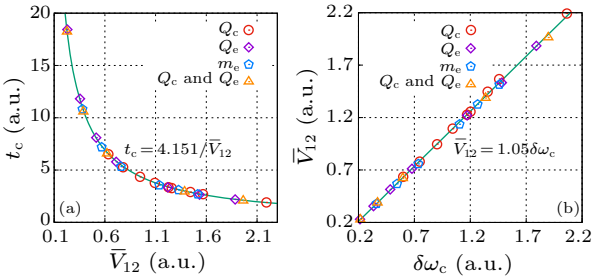

Fig. 4. The characteristic time of the TPDI varies with the Coulomb interaction energy and the Coulomb interaction energy varies with the spectral width for different cases: (1) the open circles represent the change of the nucleus charge; (2) the open diamonds represent the change of the electron charge; (3) the open pentagons represent the simultaneous change of the nucleus and electron charge; (4) the open triangles represent the change of the electron mass. The solid line is the fitting curve for the data points. The laser pulse has a Gaussian envelope around the peak intensity of 1×10^{14} W/cm^2.

Figure 4(a) shows that the characteristic time of the TPDI process varies with the Coulomb interaction energy of the ground state for the cases: (1) the change of the nucleus charge, (2) the change of the electron charge, (3) the simultaneous change of the nucleus and the electron charge, and (4) the change of the electron mass. We can find that the characteristic time t_c of the TPDI process is inversely proportional to the Coulomb interaction energy $\overline{V}_{12}$ of the ground state, i.e., there is a dynamic characteristic relationship $t_c\overline{V}_{12} \approx 4.1$ in the TPDI process. We can also see that the dynamic characteristic relationship is true in the correlated systems mentioned above from another perspective, in other words, the dynamic characteristic relationship in the TPDI process is universal. Moreover, inspired by the relationship $\delta\omega\tau = 4\sqrt{2}\ln 2$ between the spectral width (full width at half maximum in the frequency domain) $\delta\omega$ and the pulse duration τ of the Gaussian pulse, we replace the characteristic time t_c with $4\sqrt{2}\ln 2/\delta\omega_c$ in the dynamic characteristic relationship, and obtain the relationship between the Coulomb interaction energy $\overline{V}_{12}$ of the ground state and spectral width $\delta\omega_c$ is $\overline{V}_{12}/\delta\omega_c = 1.05$ as shown in Fig. 4(b), where the parameter of 1.05 is a dimensionless quantity.

In conclusion, we have investigated the TPDI process of the various helium-like three-body correlated systems by numerically solving the time-dependent Schrödinger equation. It is found that the dynamic characteristic relationship $t_c\overline{V}_{12} \approx 4.1$ in the TPDI process is universal. We also illustrate that the characteristic time defined here is clearly different from the time delay in Ref. [3], and the dynamic relationship may help us to understand the ultrafast TPDI processes more deeply.

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References

[1] Peng L Y, Jiang W C, Geng J W, Xiong W H and Gong Q H 2015 *Phys. Rep.* **575** 1
[2] Foumouo E, Antoine Ph, Bachau H and Piraux B 2008 *New J. Phys.* **10** 025017
[3] Feist J, Pazourek R, Nagele S, Persson E, Schneider B I, Collins L A and Burgdörfer J 2009 *J. Phys. B: At. Mol. Opt. Phys.* **42** 134014
[4] Pazourek R, Feist J, Nagele S and Burgdörfer J 2012 *Phys. Rev. Lett.* **108** 163001
[5] Jiang W C, Nagele S and Burgdörfer J 2017 *Phys. Rev. A* **96** 053422
[6] Ossiander M, Siegrist F, Shirvanyan V, Pazourek R, Sommer A, Latka T, Guggenmos A, Nagele S, Feist J, Burgdörfer, Kienberger J R and Schultze M 2017 *Nat. Phys.* **13** 280
[7] Hu S X 2018 *Phys. Rev. A* **97** 013414
[8] Martin L, Bello R Y, Hogle C W, Palacios A, Tong X M, Sanz-Vicario J L, Jahnke T, Schöffler M, Dörner R, Weber Th, Martín F, Kapteyn H C, Murnane M M and Ranitovic P 2018 *Phys. Rev. A* **97** 062508
[9] Xiao Z L, Quan W, Xu S P, Yu S G, Lai X Y, Chen J and Liu X J 2020 *Chin. Phys. Lett.* **37** 043201
[10] Zhang B, Zhao J and Zhao Z X 2018 *Chin. Phys. Lett.* **35** 043201
[11] Ye D, Li M, Fu L, Liu J, Gong Q, Liu Y and Ullrich J 2015 *Phys. Rev. Lett.* **115** 123001
[12] Li F, Jin F C, Yang Y J, Chen J, Yan Z C, Liu X J and Wang B B 2019 *J. Phys. B: At. Mol. Opt. Phys.* **52** 195601
[13] Levin F S and Micha D A 1994 *Long-Range Casimir Forces: Theory and Recent Experiments on Atomic Systems* (New York: Springer Science & Business Media)
[14] Bachau H, Cormier E, Decleva P and Martín F 2001 *Rep. Prog. Phys.* **64** 1815
[15] Shi T Y, Bao C G and Li B W 2001 *Commun. Theor. Phys.* **35** 195
[16] Joachain C J, Kylstra N J and Potvliege R M 2012 *Atoms in Intense Laser Fields* (Cambridge: Cambridge University Press)
[17] Shampine L F and Gordon M K 1975 *Computer Solution of Ordinary Differential Equations: The Initial Value Problem* (San Francisco: Freeman)
[18] Peng L Y and Gong Q H 2010 *Comput. Phys. Commun.* **181** 2098
[19] Liu A H and Thumm U 2014 *Phys. Rev. A* **89** 063423
[20] Riso L A and Shukla P K 2008 *J. Plasma Phys.* **74** 1
[21] Gregory C 1953 *Phys. Rev.* **92** 1554