

Dynamic solitons for the perturbed derivative nonlinear Schrödinger equation in nonlinear optics

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Abstract

Dynamic solitons for a perturbed derivative nonlinear Schrödinger equation in nonlinear optics are presented for the first time in this paper. The analytic one-soliton solution for the perturbed derivative nonlinear Schrödinger equation is obtained with the Hirota method. The stable transmission soliton is observed and the influences of third-order dispersion and nonlinear coefficients are discussed. The characteristics and properties of solitons are analyzed and the stability analysis for the solitons is made. The salient features of the solitons reveal the possibility for the stable transmission of pulses in nonlinear optics.

Keywords: ultrafast processes, optical solitons, optical propagation

(Some figures may appear in colour only in the online journal)

1. Introduction

Solitons are intrinsically nonlinear and exhibit shape-changing collisions, which find immense applications in the all-optical logic gates and optical systems [1–6]. Also solitons can be used to carry logical information for communication purposes [7–10] and can be exploited for quantum computations [11].

The dynamics of solitons (optical pulses) in nonlinear optics can be modeled by the perturbed derivative nonlinear Schrödinger (DNLS) equation [12]:

$$i \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial z^2} + im_1 \frac{\partial(|u|^2 u)}{\partial z} = im_2 \frac{\partial^3 u}{\partial z^3}, \quad (1)$$

where $u(z, t)$ is the envelope of solitons. t and z account for the propagation direction coordinates and time coordinates. m_1 and m_2 present the nonlinear and third-order dispersion effects and they are both real constants. For equation (1), when $m_2 = 0$, the equation is reduced to a completely integrable DNLS equation. The appropriate inverse scattering problem has been solved and the one-soliton solution has been

obtained [13]. By use of Hirota's method, exact N-soliton solutions have been obtained for equation (1) with $m_2 = 0$ [14, 15]. The DNLS equation with constant potential, as a model for the wave propagation on a discrete nonlinear transmission line has been considered and some exact soliton and elliptic solutions have been constructed [16]. Moreover, the propagation dynamics of EM solitons in a weak ferromagnet have been investigated and the collision of EM solitons has been established via Hirota's method [12].

However, the dynamic stable transmission of the soliton in nonlinear optics has not been reported. In this paper, we will study the dynamic stable transmission of the soliton in nonlinear optics. The one-soliton solutions will be obtained. By selecting the proper parameters of third-order dispersion and a nonlinear coefficient, soliton behaviors will be presented.

This paper will be structured as follows. In section 2, analytic one-soliton solutions will be obtained. In section 3, soliton behaviors will be analyzed and stability analysis will be made. Finally, our conclusions will be made in section 4.

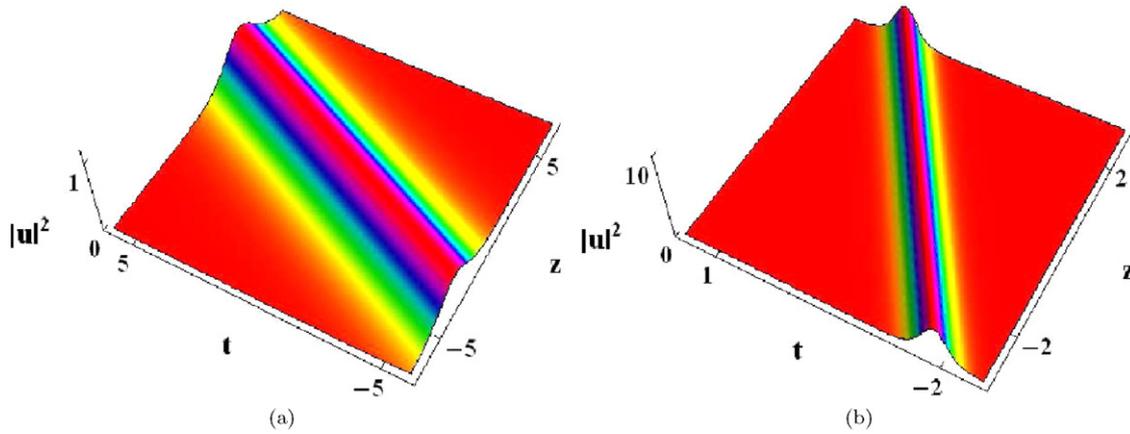


Figure 1. Transmission of the soliton in nonlinear optics. Parameters are: $k_{11} = 0.31, k_{12} = -1.2, k_{22} = 0.38$ with (a) $a_{11} = -0.56, m_1 = 0.91$ and $m_2 = -0.65$; (b) $a_{11} = 2.0, m_1 = 0.88$ and $m_2 = -0.75$.

2. Analytic soliton solutions

In order to construct the one-soliton solutions, we perform the dependent variable transformation [17–21]

$$u(z, t) = \frac{h(z, t)}{f(z, t)}, \quad (2)$$

where $h(z, t)$ is a complex differentiable function and $f(z, t)$ is a real one. With the transformation, the resulting bilinear forms for equation (1) can be derived as

$$i D_t h \cdot f + D_z^2 h \cdot f - i m_2 D_z^3 h \cdot f = 0, \quad (3)$$

$$m_2 D_z^2 f \cdot f + m_1 h^* h = 0, \quad (4)$$

$$i m_2 D_z h^* \cdot h + h^* h = 0. \quad (5)$$

D_z and D_t [22] are the Hirota’s bilinear operators, which can be defined by

$$D_z^m D_t^n (H \cdot F) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'} \right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n H(z, t) F(z', t') \Big|_{z'=z, t'=t}, \quad (6)$$

where m and n are the positive integers, H is the function of z and t and F is the function of the formal variables z' and t' .

To solve bilinear forms (3)–(5), $h(z, t)$ and $f(z, t)$ can be expanded with respect to the following power series expansions:

$$h(z, t) = \varepsilon h_1(z, t), \quad f(z, t) = 1 + \varepsilon^2 f_2(z, t), \quad (7)$$

where ε is a formal expansion parameter, $h_1(z, t)$ and $f_2(z, t)$ are the differentiable functions. Substituting expression (7) into bilinear forms (3)–(5) and equating coefficients of the same powers of ε to zero yield the recursion relations for $h_1(z, t)$ and $f_2(z, t)$. Then, the analytic one-soliton solutions for equation (1) can be obtained.

To obtain those soliton solutions, we assume that

$$h_1(z, t) = \exp(\theta_1) = \exp[(a_{11} + i a_{12})z + (b_{11} + i b_{12})t + k_{11} + i k_{12}], \quad (8)$$

where a_{1j} ’s, b_{1j} ’s and k_{1j} ’s ($j = 1, 2$) are the real constants. Substituting $h(z, t)$ into bilinear forms (3)–(5) and collecting the coefficient of ε , we get the constraints on the parameters:

$$\begin{aligned} b_{11} &= m_2 a_{11}^3 - 2a_{11} a_{12} - 3m_2 a_{11} a_{12}^2, \\ b_{12} &= a_{11}^2 - a_{12}^2 + 3m_2 a_{12} a_{11}^2 - m_2 a_{12}^3. \end{aligned} \quad (9)$$

Substituting $h(z, t)$ into bilinear forms (3)–(5) and collecting the coefficient of ε^2 , we obtain

$$f_2(z, t) = A e^{\theta_1(z, t) + \theta_1^*(z, t)} \quad \text{with} \quad A = \frac{-m_1}{8m_2 a_{11}^2}. \quad (10)$$

We substitute $h_1(z, t)$ and $f_2(z, t)$ into bilinear forms (3)–(5) and collect the coefficient of ε^3 , the coefficients of them are equal to zero with $a_{12} = -1/(2m_2)$. Without loss of generality, we set $\varepsilon = 1$ and the analytic one-soliton solutions can be expressed as

$$\begin{aligned} u(z, t) &= \frac{h(z, t)}{f(z, t)} = \frac{h_1(z, t)}{1 + f_2(z, t)} \\ &= \frac{1}{2\sqrt{A}} \operatorname{sech}(a_{11}z + b_{11}t + k_{11} + \ln\sqrt{A}) \\ &\quad \times \exp[i(a_{12}z + b_{12}t + k_{12})], \end{aligned} \quad (11)$$

where θ_1 and A are defined in expressions (9) and (10). For the purpose of verifying the solution (11), we substitute it into equation (1) and solution (11) can satisfy equation (1).

3. Discussion

In expression (11), we assume that $k_{11} = 0.31, k_{12} = -1.2, k_{22} = 0.38, a_{11} = -0.56, m_1 = 0.91$ and $m_2 = -0.75$. The soliton propagation can be obtained as shown in figure 1(a). Changing those parameters, we can increase the soliton amplitude and adjust the soliton velocity in figure 1(b). Because the soliton amplitude is related with $1/\sqrt{A}$. When the values of a_{11} and m_1 increase and the value of m_2 decreases, which results

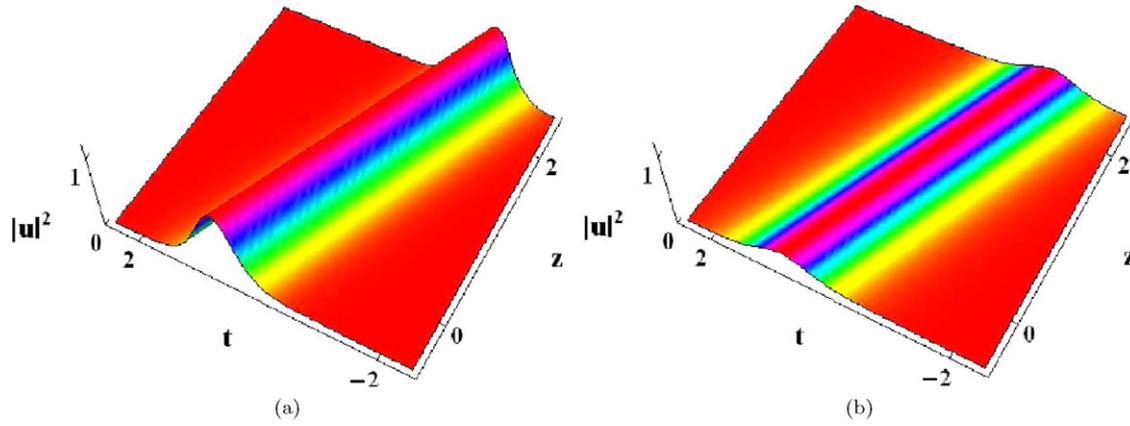


Figure 2. Transmission of the soliton in nonlinear optics with the same parameters as those given in figure 1, but with (a) $a_{11} = -1.5$, $m_1 = -1.4$ and $m_2 = 0.28$; (b) $a_{11} = -1.1$, $m_1 = -1.9$ and $m_2 = 0.19$.

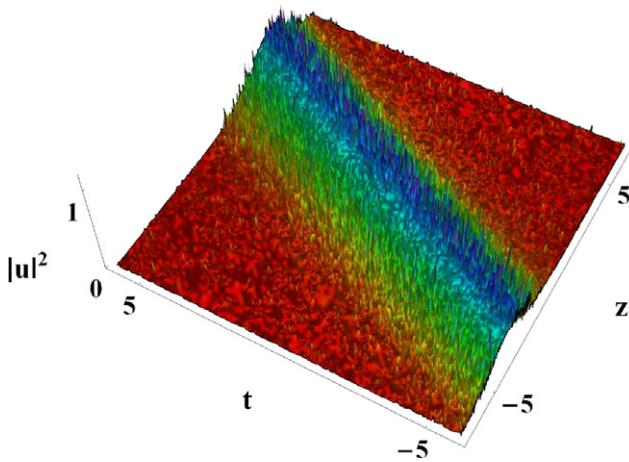


Figure 3. Stable transmission of the soliton in nonlinear optics with the same parameters as those given in figure 1(a), but with embedded white noise.

in the decreasing of the value of A . Thus, the soliton amplitude increases in figure 1(b).

If the third-order dispersion parameter m_2 is positive and the nonlinear dispersion parameter m_1 is negative, the material properties will be changed, which will result in changes of the propagation direction of solitons in the material. Similarly, the soliton amplitude decreases when the value of $\frac{-m_1}{8m_2a_{11}^2}$ increases. For example, the value of $\frac{-m_1}{8m_2a_{11}^2}$ is about 0.28 in figure 2(a), but is about 1.03 in figure 2(b). Thus, the soliton amplitude in figure 2(a) is bigger than that in figure 2(b). Therefore, we can adjust the amplitude and propagation directions of solitons through changing the corresponding parameters. These results are helpful in the study of soliton propagation in nonlinear optics.

The bright soliton shown in figure 1(a) is stable with a perturbation on m_1 and m_2 . Furthermore, the stability is analyzed with embedded white noise as figure 3 shown. The intensity of the white noise is 0.1 and the transmission of the soliton is stable with some perturbations in the amplitude.

4. Conclusions

Dynamic stable transmission of solitons in nonlinear optics have been investigated for the first time in this paper. By virtue of Hirota method, analytic one-soliton solution (12) for equation (1) has been obtained. Through changing the third-order dispersion coefficient m_2 and nonlinear coefficient m_1 , we can adjust the soliton amplitude and propagation directions. and the stability analysis for solitons has demonstrated that the transmission of solitons is stable in nonlinear optics. Results in this paper are useful for studying the soliton characteristics and properties in nonlinear optics.

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