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Long-distance interactions between optical solitons with an oscillating structure

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Abstract
In this letter, long-distance interactions between optical solitons with an oscillating structure are investigated. Analytic two-soliton solutions for the variable-coefficient nonlinear Schrödinger equation are obtained. Different from the elastic interactions reported previously, the interaction solitons are accompanied with an oscillating structure during their interactions. Reasons for long-distance interactions are discussed, and influences of the corresponding parameters are analyzed. Those studies may provide a new insight into the soliton interactions.

Keywords: solitons, soliton interactions, analytic solutions, variable-coefficient nonlinear Schrödinger equation

(Some figures may appear in colour only in the online journal)

1. Introduction

Various aspects of optical solitons have been the subject of intense investigations in recent decades, as a result of both their fundamental relevance and potential versatility in applications ranging from data transmission to readdressing or switching [1–9]. Optical solitons have the tendency to maintain their shapes during the propagation due to the balance between the group velocity dispersion (GVD) and nonlinearity [10, 11]. One prominent theme of research in optical solitons is the soliton interaction [12–16]. When two optical solitons are mutually coherent, the interaction force between solitons can be attractive or repulsive, depending on their relative phase, and the properties they exhibit are normally associated with particles [17].

Traditionally, soliton interactions are usually considered to be elastic [18]. After soliton interactions, there is no change in physical quantities such as their amplitudes, velocities and shapes [18]. However, depending upon the individual pulse width, inter-pulse spacing and loss in optical fibers, co-propagating solitons do interact and share energy [11]. It is therefore necessary to investigate soliton interactions before implementing them in high speed optical communication systems. To overcome that problem, several effective methods have been proposed, and considerable theoretical and experimental research has been carried out on soliton interactions [10, 19].

The nonlinear Schrödinger (NLS) equation can be used to study the properties and features of soliton interactions in nonlinear optics. The following variable-coefficient NLS (vcNLS) equation is under investigation in this letter [10]:

$$\mathrm{i}\frac{\partial u}{\partial z}+\mathrm{i}\beta_1(z)\frac{\partial u}{\partial t}+\beta_2(z)\frac{\partial^2 u}{\partial t^2}+\gamma(z)|u|^2u=0, \qquad (1)$$

where $u(z, t)$ is the temporal envelope of optical solitons. t is the time in the moving coordinate system, z is the longitudinal coordinate. $\beta_1(z)$ is the reciprocal of the group velocity, $\beta_2(z)$ represents the GVD coefficient, and $\gamma(z)$ is the nonlinearity coefficient. When $\beta_1(z) = 0$, equation (1) can be reduced into the standard vcNLS equation, and elastic interactions between optical solitons have been studied for the standard vcNLS equation [10].

However, soliton interactions in equation (1) have not been studied. With the aid of the transformation in [20, 21], equation (1) usually becomes the standard NLS equation. Researchers have studied the solutions of the standard NLS equation. However, they have not discussed the influence of $\beta_1(z)$ on soliton interactions with the final solution versus t.

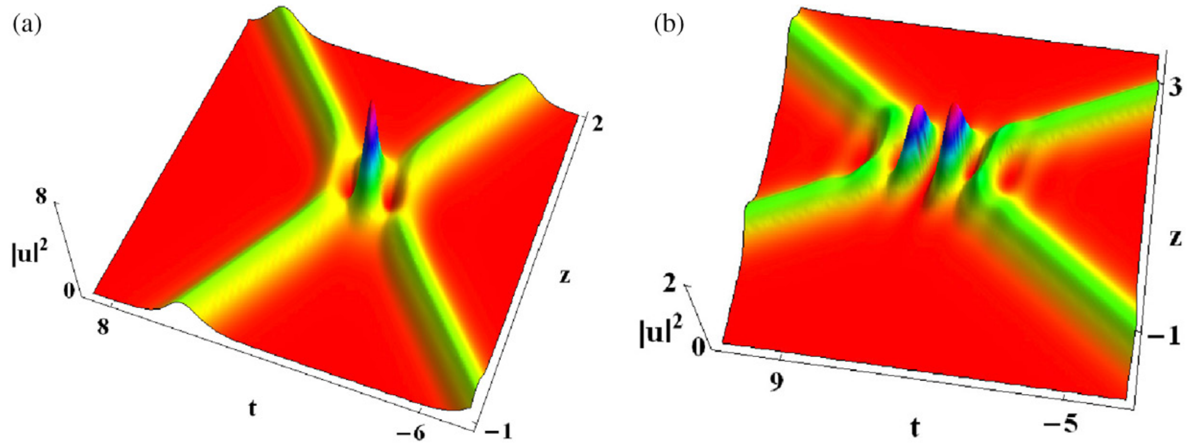

Figure 1. Soliton interactions with an oscillating structure. The parameters are $M = 1$, $\beta_1(z) = 1$, $\gamma(z) = 2$, $k_{11} = -1$, $b_{12} = 1$ with (*a*) $k_{12} = 0$, $k_{21} = -1$, $k_{22} = 0$, $b_{11} = 1$, $b_{21} = -1$, $b_{22} = -1$, (*b*) $k_{12} = 1$, $k_{21} = 2$, $k_{22} = 1$, $b_{11} = 0.5$, $b_{21} = -0.5$ and $b_{22} = -2$.

In this paper, we will directly analyze the influence of $\beta_1(z)$ on soliton interactions using the solution forms. With $\beta_1(z)$, we will present a new type of soliton interaction. The interaction solitons are accompanied with an oscillating structure during their interactions. We find that their interaction dynamics are different from the elastic interactions reported previously [21]. The interaction strength can be controlled. More strikingly, the interaction can change from attractive to repulsive by changing the corresponding parameters, or vice versa.

This letter will be structured as follows. In section 2, the analytic two-soliton solutions for equation (1) will be obtained. In section 3, the properties and features of interaction solitons will be discussed, and the influence of the corresponding parameters on soliton interactions will be analyzed. Finally, our conclusions will be given in section 4.

2. Analytic two-soliton solutions

At first, the dependent variable transformation can be introduced as [20–23]

$$u(z, t) = \frac{g(z, t)}{f(z, t)}, \tag{2}$$

where $g(z, t)$ is a complex differentiable function, and $f(z, t)$ is a real one (equation (1)). With symbolic computation, the bilinear forms for equation (1) are obtained as

$$\mathrm{i}D_z g\cdot f + \mathrm{i}\beta_1(z)\, D_t g\cdot f + \beta_2(z)\, D_t^2 g\cdot f = 0, \tag{3}$$

$$\beta_2(z)\, D_t^2 f\cdot f - \gamma(z)\, g g^* = 0, \tag{4}$$

where the asterisk denotes the complex conjugate. D_z and D_t [24] are Hirota's bilinear operators, and are defined by

$$D_z^m D_t^l (a\cdot b) = \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^l a(z, t)\, b(z', t')\Big|_{z'=z,\, t'=t}, \tag{5}$$

where a and b are functions of z and t. m and l are positive integers. With the following power series expansions for $g(z, t)$ and $f(z, t)$:

$$g(z, t) = \varepsilon g_1(z, t) + \varepsilon^3 g_3(z, t) + \varepsilon^5 g_5(z, t) + \cdots, \tag{6}$$

$$f(z, t) = 1 + \varepsilon^2 f_2(z, t) + \varepsilon^4 f_4(z, t) + \varepsilon^6 f_6(z, t) + \cdots, \tag{7}$$

where ε is a formal expansion parameter, bilinear forms (3)–(4) can be solved. Substituting expressions (6)–(7) into bilinear forms (3)–(4) and equating coefficients of the same powers of ε to zero yield the recursion relations for $g_n(z, t)$ and $f_n(z, t)$. Then, the analytic two-soliton solutions for equation (1) can be derived.

To obtain the analytic two-soliton solutions for equation (1), we assume

$$g(z, t) = g_1(z, t) + g_3(z, t), \quad f(z, t) = 1 + f_2(z, t) + f_4(z, t), \tag{8}$$

where

$$\begin{aligned} g_1(z, t) &= \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2}, \ \theta_1 = a_1(z)\, z \\ &\quad + b_1 t + k_1 = [a_{11}(z) + \mathrm{i}a_{12}(z)]\, z \\ &\quad + (b_{11} + \mathrm{i}b_{12})\, t + k_{11} + \mathrm{i}k_{12}, \ \theta_2 = a_2(z)\, z \\ &\quad + b_2 t + k_2 = [a_{21}(z) + \mathrm{i}a_{22}(z)]\, z \\ &\quad + (b_{21} + \mathrm{i}b_{22})\, t + k_{21} + \mathrm{i}k_{22}, \end{aligned} \tag{9}$$

with b_{j1}, b_{j2}, k_{j1} and $k_{j2} (j = 1, 2)$ being real constants. $a_{j1}(z)$ and $a_{j2}(z)$ are the differentiable functions to be determined. With $g_1(z, t)$, and collecting the coefficient of ε in equation (3), we obtain the constraints on $a_{j1}(z)$ and $a_{j2}(z)$ as

$$a_{j1}(z) = \frac{1}{z}\int \left[-b_{j1}\beta_1(z) - 2b_{j1}b_{j2}\beta_2(z) \right] \mathrm{d}z,$$

$$a_{j2}(z) = \frac{1}{z}\int \left[b_{j1}^2\beta_2(z) - b_{j2}\beta_1(z) - b_{j2}^2\beta_2(z) \right] \mathrm{d}z.$$

Substituting $g_1(z, t)$ into equation (4), and collecting the coefficient of ε^2, we yield

$$f_2(z, t) = A_1 \mathrm{e}^{\theta_1+\theta_1^*} + A_2 \mathrm{e}^{\theta_2+\theta_2^*} + A_3 \mathrm{e}^{\theta_1+\theta_2^*} + A_4 \mathrm{e}^{\theta_2+\theta_1^*} \tag{10}$$

with

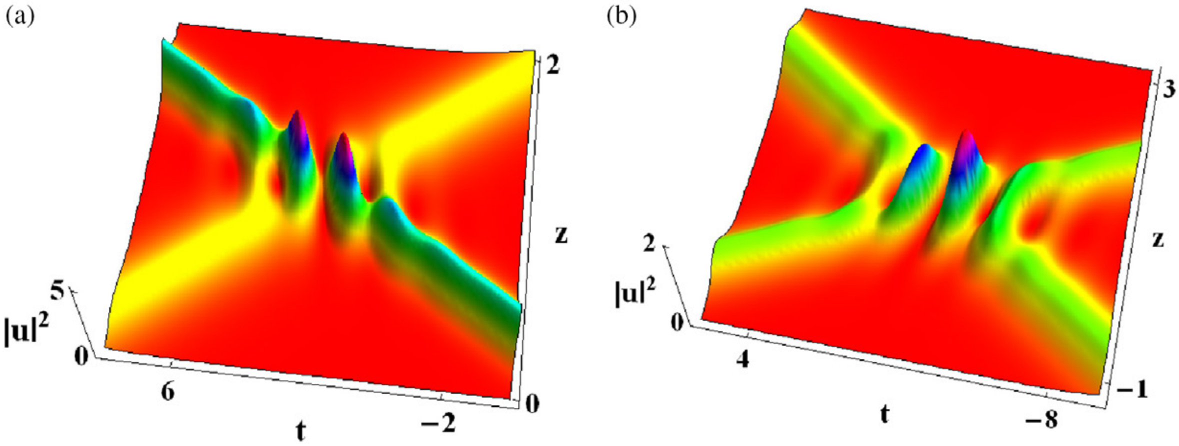

Figure 2. Soliton interactions with an oscillating structure with the same parameters as those given in figure 1(*b*), but with (*a*) $b_{11} = 1$, $b_{12} = 2$, $b_{21} = -0.6$, (*b*) $k_{11} = 1$, $k_{12} = -1$ and $k_{21} = -2$.

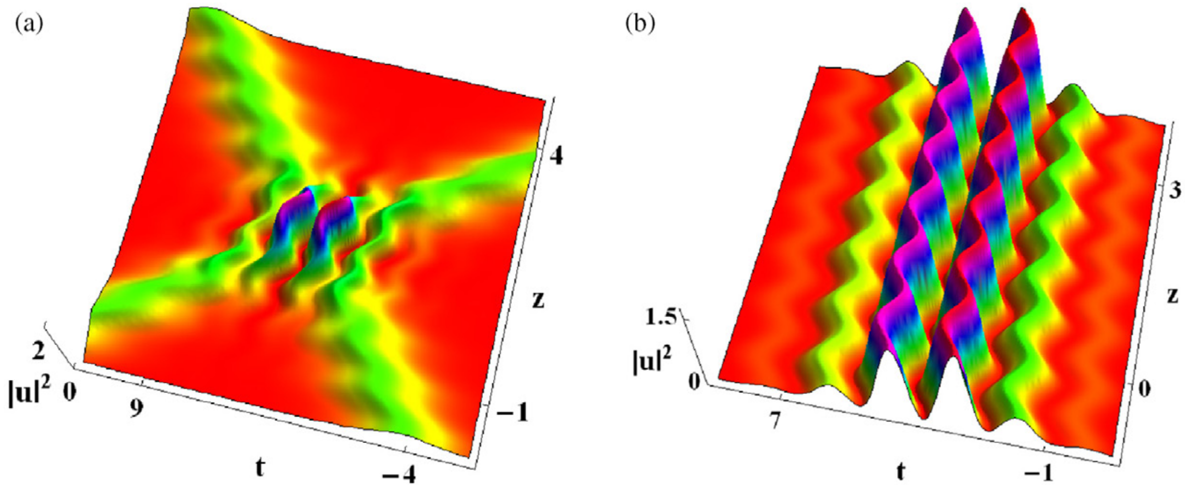

Figure 3. Soliton interactions with an oscillating structure with the same parameters as those given in figure 1(*b*), but with $\beta_1(z) = 2\cos(8z)$, (*a*) $\gamma(z) = 1$ and (*b*) $\gamma(z) = 0.01$.

$$A_1 = \frac{M}{8b_{11}^2},\quad A_2 = \frac{M}{8b_{21}^2},\quad A_3 = \frac{M}{2\left(b_1 + b_2^*\right)^2},$$
$$A_4 = \frac{M}{2\left(b_1^* + b_2\right)^2},\quad \gamma(z) = M\beta_2(z),$$

and M as an arbitrary constant.

In order to obtain $g_3(z, t)$, we substitute $g_1(z, t)$ and $f_2(z, t)$ into equation (4), and obtain

$$g_3(z,\ t) = B_1 e^{\theta_1+\theta_2+\theta_1^*} + B_2 e^{\theta_1+\theta_2+\theta_2^*}, \tag{11}$$

with

$$B_1 = \frac{M(b_1 - b_2)^2}{8b_{11}^2\left(b_1^* + b_2\right)^2},\quad B_2 = \frac{M(b_1 - b_2)^2}{8b_{21}^2\left(b_1 + b_2^*\right)^2}.$$

Finally, we can obtain $f_4(z, t)$ as

$$f_4(z,\ t) = B_3 e^{\theta_1+\theta_2+\theta_1^*+\theta_2^*}, \tag{12}$$

with

$$B_3 = \frac{M^2[(b_{11} - b_{21})^2 + (b_{12} - b_{22})^2\,]^2}{64 b_{11}^2 b_{21}^2\left(b_1 + b_2^*\right)^2\left(b_1^* + b_2\right)^2}.$$

Without loss of generality, we write the analytic two-soliton solutions for equation (1) as

$$u(z,\ t) = \frac{g(z,\ t)}{f(z,\ t)} = \frac{g_1(z,\ t) + g_3(z,\ t)}{1 + f_2(z,\ t) + f_4(z,\ t)}. \tag{13}$$

3. Discussion

According to solutions (13), we can obtain the soliton interactions with the parameters $M = 1$, $\beta_1(z) = 1$, $\gamma(z) = 2$, $k_{11} = -1$, $b_{12} = 1$, $k_{12} = 0$, $k_{21} = -1$, $k_{22} = 0$, $b_{11} = 1$, $b_{21} = -1$ and $b_{22} = -1$ in figure 1(*a*). That phenomenon is the classic soliton interaction. The interactions between solitons are elastic, and the soliton properties do not change after interactions. However, when the values of b_{j1}, b_{j2}, k_{j1} and k_{j2} change, two solitons are accompanied with an oscillating structure during their interactions as shown in figure 1(*b*).

The long-distance interactions between solitons being accompanied with an oscillating structure are due to the existence of the reciprocal of the group velocity $\beta_1(z)$.

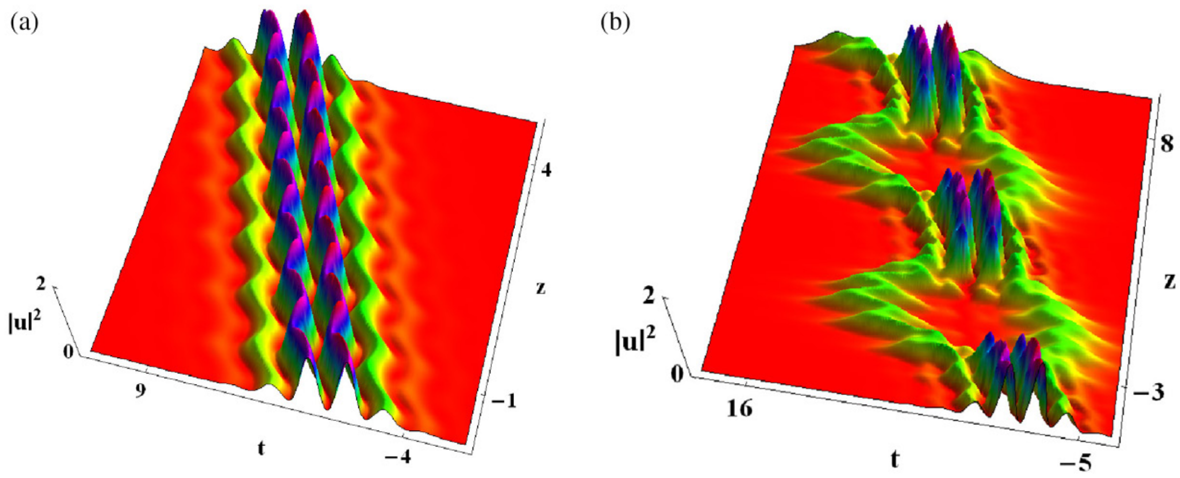

Figure 4. Soliton interactions with an oscillating structure with the same parameters as those given in figure 1(b), but with $\beta_1(z) = 1$, (a) $\gamma(z) = 2\cos(6z)$, (b) $\gamma(z) = 2\cos(8z) + \sin(z)$.

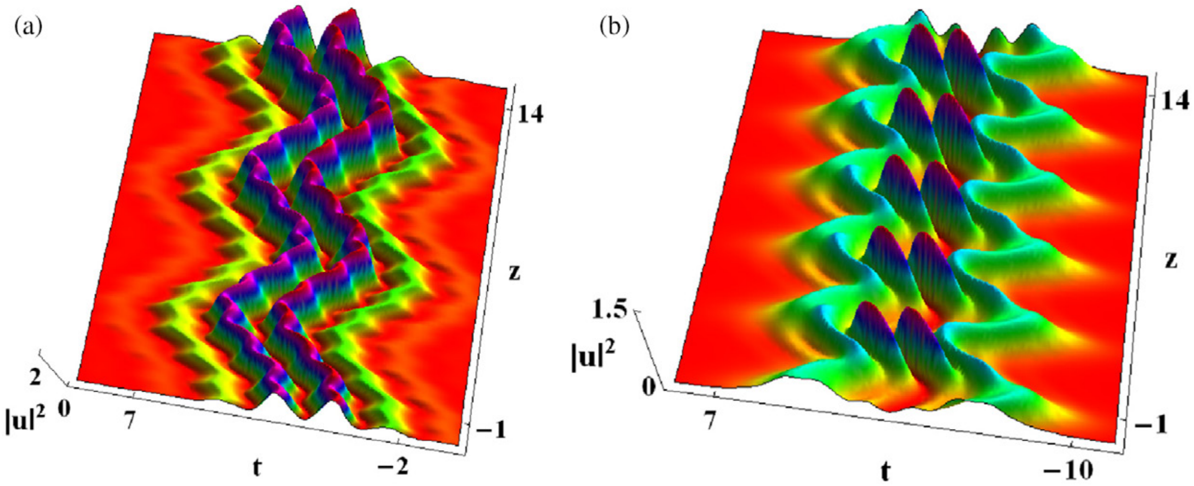

Figure 5. Soliton interactions with an oscillating structure with the same parameters as those given in figure 1(b), but with (a) $\beta_1(z) = \cos(z)$, $\gamma(z) = \cos(6z)$, (b) $\beta_1(z) = 2\cos(2z)$ and $\gamma(z) = \sin(2z)$.

Before soliton interactions, $\beta_1(z)$ results in the change of soliton phases and velocities, then the soliton oscillations are generated. When solitons interact with each other after a while, the interactions between them are weakened, and they separate in accordance with the original velocities and phases. The solitons change periodically when they interact with each other. Changing the values of b_{j1} and b_{j2} can adjust the amplitude of solitons in figure 2(a). The reason for the increasing amplitude can be explained by A_1, A_2, A_3, A_4, B_1, B_2 and B_3 in expressions (10)–(12). The soliton amplitude decreases with the decreasing of b_{21}, and the period of the soliton interactions increases. The values of k_{j1} and k_{j2} can adjust the initial phases of solitons, which can be seen in figure 2(b). Moreover, k_{j1} and k_{j2} can determine the interaction distance of solitons, which depends on the initial phase difference between solitons. When the initial phase difference between solitons is $\pi/2$, the solitons interact with the shortest distance. When the initial phase difference between solitons is 0 or π, the soliton interactions have the longest interaction distance.

In figures 1 and 2, the values of $\beta_1(z)$ and $\gamma(z)$ are constants. Due to various perturbations, the balance between the dispersion and nonlinearity is broken, and the propagation of solitons is affected. In order to maintain a well-ordered propagation of solitons, it is necessary to introduce a slow change of the fiber parameters in the longitudinal direction. Therefore, the values of $\beta_1(z)$ and $\gamma(z)$ will be assumed to be a function of z. When $\beta_1(z) = 2\cos(8z)$ and $\gamma(z) = 1$, the solitons change periodically in figure 3(a). The velocities of the solitons change in a cosine function due to the variation of $\beta_1(z)$. When we decrease the value of $\gamma(z)$, the nonlinear effect results in the change of soliton phases, and the solitons propagate in parallel in figure 3(b). Although they interact with each other, they propagate in a bound state of solitons. However, when $\beta_1(z)$ is a constant, $\gamma(z)$ is a trigonometric form, such as $\beta_1(z) = 1$ and $\gamma(z) = 2\cos(6z)$ in figure 4(a), the bound solitons are also obtained. Changing the value of $\gamma(z)$, the bound state of solitons changes as shown in figure 4(b). When $\beta_1(z)$ and $\gamma(z)$ are both functions, different bound states of the solitons are displayed in figure 5. Therefore, we can adjust the bound state of solitons through changing the values of $\beta_1(z)$ and $\gamma(z)$. At the same time, by analyzing the influences of $\beta_1(z)$ and $\gamma(z)$, we can avoid the disordered propagation of solitons when the dispersion and nonlinearity can not be balanced.

4. Conclusions

Long-distance interactions between optical solitons with an oscillating structure have been investigated in this letter. The vcNLS equation (see equation (1)), which can be used to describe the soliton propagation, has been studied analytically. The analytic two-soliton solutions (13) have been obtained. Due to the existence of the reciprocal of the group velocity $\beta_1(z)$, the interaction solitons have been accompanied with an oscillating structure during their interactions (see figures 1–5), and the influences of the parameters of b_{j1}, b_{j2}, k_{j1} and k_{j2} on soliton interactions have been discussed. The amplitude of the solitons has been adjusted through changing the values of b_{j1} and b_{j2} (see figure 2(*a*)). The soliton interactions have been changed from attractive to repulsive by changing the values of $\beta_1(z)$ and $\gamma(z)$ (see figures 4 and 5). Our studies suggest that those phenomena could be used for studying the dispersion management systems.

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