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Letter

Breathers in a hollow-core photonic crystal fiber

Wen-Jun Liu[1,2], Hai-Nian Han[1,3], Long Zhang[1], Rui Wang[1], Zhi-Yi Wei[1,3] and Ming Lei[1,2]

[1] Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China
[2] School of Science, PO Box 122, Beijing University of Posts and Telecommunications, Beijing 100876, People's Republic of China

E-mail: hnhan@iphy.ac.cn and zywei@iphy.ac.cn

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Abstract
In this work breathers are obtained in a hollow-core photonic crystal fiber (HC-PCF) for the first time. The nonlinear Schrödinger equation describing the propagation of pulses in a HC-PCF is investigated using the Hirota bilinear method and the auxiliary function method. Analytic breather solutions are derived by an appropriate choice of parameters. Dynamical behavior of breathers is exhibited, and the influences of different parameters on the characteristics of breathers are discussed. The presented results could be used in fiber lasers, nonlinear optics and Bose–Einstein condensates.

Keywords: breathers, hollow-core photonic crystal fibers, Hirota bilinear method, auxiliary function method, analytic breather solutions

(Some figures may appear in colour only in the online journal)

1. Introduction

Photonic crystal fibers (PCFs) are optical fibers based on the properties of photonic crystals [1]. Because of their ability to confine light in hollow cores or with confinement characteristics not possible in conventional optical fibers, PCFs possess numerous unusual properties, including highly tunable dispersion, nonlinearity and single mode operation at all wavelengths. Those properties are of fundamental importance for applications in fiber-optic communications, fiber lasers, nonlinear devices, high-power transmission and highly sensitive gas sensors [2–5]. More specific categories of PCFs include photonic bandgap fibers, holey fibers, hole-assisted fibers and Bragg fibers. Among these, some attention has been paid to hollow-core photonic crystal fibers (HC-PCFs) [6–9].

HC-PCFs, which guide light due to the presence of a photonic bandgap, represent a new generation of low-loss transmission fibers. They enable high-power light delivery in a single spatial mode [10]. HC-PCFs have also been considered as suitable 'hosts' for overcoming the difficulties caused by nonlinear interactions between laser light and low-density gas media [11]. These advantages lead to many fascinating applications for HC-PCFs, such as in sensors and nonlinear optics in which the gas is introduced into the core region [12, 13].

In this paper, the nonlinear dynamics of HC-PCFs will be studied analytically. The propagation of pulses in a HC-PCF can be described by the following nonlinear Schrödinger (NLS) equation [14, 15]:

$$\mathrm{i}\,\partial_\xi\psi - \frac{1}{2}\beta_2\,\partial_\tau^2\psi - \frac{\mathrm{i}}{6}\beta_3\,\partial_\tau^3\psi + |\psi|^2\psi - \tau_R\,\psi\,\partial_\tau|\psi|^2 - \eta\,\psi\int_{-\infty}^{\tau}|\psi|^2\,\mathrm{d}\tau' = 0, \quad (1)$$

where $\psi(\xi,\tau)$ is the normalized electric-field envelope, ξ is the longitudinal coordinate along the HC-PCF and τ is the

[3] Authors to whom any correspondence should be addressed.

time coordinate in a reference frame that moves with the pulse group velocity. β_2 represents the group-velocity dispersion coefficient and β_3 is the third-order dispersion coefficient. $\tau_R \equiv \int_0^\infty \tau'[(1-\rho)\widetilde{\delta}(\tau') + \rho\, h(\tau')]\, t_0\, d\tau'$, where $\widetilde{\delta}(\tau')$ is the Dirac delta function, t_0 is the input pulse duration, ρ is the relative strength of the non-instantaneous Raman nonlinearity and $h(\tau')$ is the causal Raman response function of the gas [16, 17]. $\eta \equiv k_0 \widetilde{\sigma} t_0 \omega_T^2/(2A_{\rm eff}\, \gamma_K \omega_0^2)$, where $k_0 = \omega_0/c$ with c being the speed of light and ω_0 is the input pulse central frequency, $\widetilde{\sigma}$ is related to two constants [14], ω_T is the maximum plasma frequency, $A_{\rm eff}$ is the effective mode area and γ_K is the nonlinear Kerr coefficient of the gas.

For equation (1), solitons, being a type of localized nonlinear excitation, have been studied extensively in HC-PCFs [14, 15, 18]. An analytic study on controlling soliton dynamics in a HC-PCF has been presented and the features and properties of solitons have been discussed in [18]. It has been theoretically shown [15] that photoionization leads to a constant acceleration of solitons in the time domain with a continuous shift to higher frequencies, limited only by ionization loss. By applying the Gagnon–Bélanger gauge transformation, stationary negative-slope two-peak inverted gravity-like soliton solutions have been obtained for pulses propagating in a HC-PCF filled by Raman-inactive gases [14]. Moreover, unconventional long-range nonlocal interactions between temporally distant solitons, unique to gas plasma systems, have been predicted and studied [14].

A more widespread class of localized nonlinear excitations, which are perhaps even more important than solitons in nonlinear optics and Bose–Einstein condensates (BECs), are breathers [19]. Breathers need practically no activation energy, and can bridge the gap between highly nonlinear modes and linear phonon modes [19]. Furthermore, the internal degree of freedom of breathers increases their potential to describe physical phenomena. Thus, it is necessary to investigate breathers in HC-PCFs.

To the best of our knowledge no studies have explicitly characterized breathers in a HC-PCF described by equation (1). The present paper has the goal of demonstrating the existence of breathers in HC-PCFs. Here we focus on equation (1), which will be studied by means of the Hirota bilinear method and the auxiliary function method. We present for the first time analytic breather solutions for equation (1), and the influences on these breathers will be discussed. Our results may have an important role in the research of some physical phenomena such as fiber lasers, nonlinear optics and BECs.

The paper will be structured as follows. In section 2 we present the analytic breather solutions for equation (1). In section 3 we study the features and properties of breathers, and analyze the influences on breathers. Finally in section 4 we summarize our findings and present our conclusions.

2. Bilinear forms and analytic breather solutions for equation (1)

First we introduce the dependent variable transformation [20–23]

$$\psi(\xi,\tau) = \frac{g(\xi,\tau)}{f(\xi,\tau)}, \tag{2}$$

where $g(\xi,\tau)$ is a complex differentiable function and $f(\xi,\tau)$ is a real one. After some symbolic manipulations, bilinear forms with an auxiliary function $s = s(\xi,\tau)$ can be obtained for equation (1) as

$$\tfrac{1}{2}\,\beta_2\, D_\tau^2\, ff + g\, g^* = 0, \tag{3}$$

$$D_\tau\, gg^* - s\, f = 0, \tag{4}$$

$$\mathrm{i}\, D_\xi\, gf - \tfrac{1}{2}\,\beta_2\, D_\tau^2\, gf - \frac{\mathrm{i}}{6}\,\beta_3\, D_\tau^3\, gf + \eta\, \beta_2\, g\, f_\tau + \tau_R\, g\, s = 0 \tag{5}$$

with $\beta_3 = 2\,\mathrm{i}\,\tau_R\,\beta_2$.

Here, Hirota's bilinear operators D_ξ and D_τ [24] are defined by

$$D_\xi^m\, D_\tau^n(ab) = \left(\frac{\partial}{\partial\xi} - \frac{\partial}{\partial\xi'}\right)^m \left(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\tau'}\right)^n \times a(\xi,\tau)\, b(\xi',\tau')\big|_{\xi'=\xi,\ \tau'=\tau}. \tag{6}$$

The bilinear forms (3)–(5) can be solved by the following power series expansions for $g(\xi,\tau)$, $f(\xi,\tau)$ and $s(\xi,\tau)$:

$$g(\xi,\tau) = \varepsilon\, g_1(\xi,\tau) + \varepsilon^3\, g_3(\xi,\tau) + \varepsilon^5\, g_5(\xi,\tau) + s, \tag{7}$$

$$f(\xi,\tau) = 1 + \varepsilon^2\, f_2(\xi,\tau) + \varepsilon^4\, f_4(\xi,\tau) + \varepsilon^6\, f_6(\xi,\tau) + s, \tag{8}$$

$$s(\xi,\tau) = \varepsilon^2\, s_2(\xi,\tau) + \varepsilon^4\, s_4(\xi,\tau) + \varepsilon^6\, s_6(\xi,\tau) + s, \tag{9}$$

where ε is a formal expansion parameter. Substituting expressions (7)–(9) into bilinear forms (3)–(5) and equating coefficients of the same powers of ε to zero yields the recursion relations for $g_n(\xi,\tau)$, $f_n(\xi,\tau)$ and $s_n(\xi,\tau)$. Then, analytic breather solutions for equation (1) can be obtained.

To derive the analytic breather solutions, we take

$$\begin{aligned} g(\xi,\tau) &= g_1(\xi,\tau) + g_3(\xi,\tau),\\ f(\xi,\tau) &= 1 + f_2(\xi,\tau) + f_4(\xi,\tau),\\ s(\xi,\tau) &= s_2(\xi,\tau) + s_4(\xi,\tau), \end{aligned} \tag{10}$$

where

$$g_1(\xi,\tau) = \mathrm{e}^{\theta_1} + \mathrm{e}^{\theta_2}, \tag{11}$$

$$\begin{aligned} \theta_j &= a_j\, \xi + b_j\, \tau + k_j\\ &= (a_{j1} + \mathrm{i}\, a_{j2})\, \xi + (b_{j1} + \mathrm{i}\, b_{j2})\, \tau + k_{j1} + \mathrm{i}\, k_{j2} \end{aligned}$$

with a_{j1}, a_{j2}, b_{j1}, b_{j2}, k_{j1} and k_{j2} (($j = 1, 2$) are real constants). With $g_1(\xi,\tau)$, and collecting the coefficient of ε in equation (5), we can get the constraints on the parameters:

$$\begin{aligned} a_{j1} &= \tfrac{1}{3}\beta_2\tau_R b_{j1}^3 - b_{j2}\beta_2 b_{j1} - b_{j2}^2\beta_2\tau_R b_{j1},\\ a_{j2} &= -\tfrac{1}{3}\beta_2\tau_R b_{j2}^3 - \tfrac{1}{2}\beta_2 b_{j2}^2 + b_{j1}^2\beta_2\tau_R b_{j2} + \tfrac{1}{2} b_{j1}^2\beta_2. \end{aligned}$$

Substituting $g_1(\xi,\tau)$ into equations (3) and (4), and collecting the coefficient of ε^2 yields

$$\begin{aligned} f_2(\xi,\tau) &= A_1 \mathrm{e}^{\theta_1+\theta_1^*} + A_2 \mathrm{e}^{\theta_2+\theta_2^*} + A_3 \mathrm{e}^{\theta_1+\theta_2^*} + A_4 \mathrm{e}^{\theta_2+\theta_1^*},\\ s_2(\xi,\tau) &= B_1 \mathrm{e}^{\theta_1+\theta_1^*} + B_2 \mathrm{e}^{\theta_2+\theta_2^*} + B_3 \mathrm{e}^{\theta_1+\theta_2^*} + B_4 \mathrm{e}^{\theta_2+\theta_1^*} \end{aligned} \tag{12}$$

with

$$A_1 = \frac{1}{4b_{11}^2\beta_2}, \qquad A_2 = \frac{1}{4b_{21}^2\beta_2},$$
$$A_3 = \frac{1}{\left(b_1 + b_2^*\right)^2 \beta_2}, \qquad A_4 = A_3^*,$$
$$B_1 = 2\mathrm{i}b_{12}, \qquad B_2 = 2\mathrm{i}b_{22},$$
$$B_3 = b_1 - b_2^*, \qquad B_4 = b_2 - b_1^*.$$

We substitute $g_1(\xi,\tau)$, $f_2(\xi,\tau)$ and $s_2(\xi,\tau)$ into equation (5) and collect the coefficient of ε^3, yielding the expression $g_3(\xi,\tau)$ as

$$g_3(\xi,\tau) = E_1 e^{2\theta_1+\theta_1^*} + E_2 e^{2\theta_1+\theta_2^*} + E_3 e^{\theta_1+\theta_2+\theta_1^*} + E_4 e^{\theta_1+\theta_2+\theta_2^*} + E_5 e^{2\theta_2+\theta_1^*} + E_6 e^{2\theta_2+\theta_2^*} \tag{13}$$

with

$$E_1 = \frac{4\mathrm{i}b_{11}b_{12}\tau_R - \eta}{8b_{11}^3\beta_2\left(2b_1\tau_R + \mathrm{i}\right)},$$
$$E_2 = \frac{(b_1^2 - b_2^{*2})\tau_R - \eta}{\left(b_1 + b_2^*\right)^3 \beta_2\left(2b_1\tau_R + \mathrm{i}\right)},$$
$$\begin{aligned}E_3 = {} & \big(\mathrm{i}\left(b_1 - b_2\right)^2 + \eta\left(b_1 - b_2\right) - 4\eta b_{11} + \left(b_2^3 - b_1^3\right)\tau_R \\ & + 2\mathrm{i}b_{12}\tau_R\left(7b_{11}^2 - b_{12}^2\right) + b_2 b_1^*\left(b_2 - b_1^*\right)\tau_R\big) \\ & \times \left(4b_{11}^2\left(b_1^* + b_2\right)^2\beta_2\left(b_1\tau_R + b_2\tau_R + \mathrm{i}\right)\right)^{-1},\end{aligned}$$
$$\begin{aligned}E_4 = {} & \big(\mathrm{i}\left(b_2 - b_1\right)^2 + \eta\left(b_2 - b_1\right) - 4\eta b_{21} + \left(b_1^3 - b_2^3\right)\tau_R + 2\mathrm{i}b_{22}\tau_R\left(7b_{21}^2 - b_{22}^2\right) \\ & + b_1 b_2^*\left(b_1 - b_2^*\right)\tau_R\big)\big(4b_{21}^2\left(b_1 + b_2^*\right)^2 \\ & \times \beta_2\left(b_1\tau_R + b_2\tau_R + \mathrm{i}\right)\big)^{-1},\end{aligned}$$
$$E_5 = \frac{(b_2^2 - b_1^{*2})\tau_R - \eta}{\left(b_1^* + b_2\right)^3\beta_2\left(2b_2\tau_R + \mathrm{i}\right)},$$
$$E_6 = \frac{4\mathrm{i}b_{21}b_{22}\tau_R - \eta}{8b_{21}^3\beta_2\left(2b_2\tau_R + \mathrm{i}\right)}.$$

In order to obtain $f_4(\xi,\tau)$, substituting $g_1(\xi,\tau)$, $f_2(\xi,\tau)$, $s_2(\xi,\tau)$ and $g_3(\xi,\tau)$ into equation (3), and collecting the coefficient of ε^4, we get

$$f_4(\xi,\tau) = M_1 e^{2\theta_2+2\theta_2^*} + M_2 e^{\theta_1+\theta_2+2\theta_2^*} + M_3 e^{2\theta_1+2\theta_2^*} + M_4 e^{2\theta_2+\theta_1^*+\theta_2^*} + M_5 e^{\theta_1+\theta_2+\theta_1^*+\theta_2^*} + M_6 e^{2\theta_1+\theta_1^*+\theta_2^*} + M_7 e^{2\theta_2+2\theta_1^*} + M_8 e^{\theta_1+\theta_2+2\theta_1^*} + M_9 e^{2\theta_1+2\theta_1^*} \tag{14}$$

with

$$M_1 = \frac{E_6 + E_6^*}{16b_{21}^2\beta_2},$$
$$M_2 = \frac{E_4 + E_5^* + E_6^* - A_2A_3\beta_2\left(b_1 - b_2\right)^2}{\left(b_1 + b_2^* + 2b_{21}\right)^2\beta_2},$$
$$M_3 = \frac{E_2 + E_5^*}{4\left(b_1 + b_2^*\right)^2\beta_2},$$
$$M_4 = \frac{E_4^* + E_5 + E_6 - A_2A_4\beta_2\left(b_1^* - b_2^*\right)^2}{\left(b_1^* + b_2 + 2b_{21}\right)^2\beta_2},$$
$$M_6 = \frac{E_1 + E_2 + E_3^* - A_1A_3\beta_2\left(b_1^* - b_2^*\right)^2}{\left(b_1 + b_2^* + 2b_{11}\right)^2\beta_2},$$
$$\begin{aligned}M_5 = {} & \big(E_3 + E_4 + E_3^* + E_4^* - 4A_1A_2\beta_2 \\ & \times \left(b_{11} - b_{21}\right)^2 + 4A_3A_4\beta_2\left(b_{12} - b_{22}\right)^2\big) \\ & \times \left(4\left(b_{11} + b_{21}\right)^2\beta_2\right)^{-1},\end{aligned}$$
$$M_7 = \frac{E_2^* + E_5}{4\left(b_1^* + b_2\right)^2\beta_2},$$
$$M_8 = \frac{E_1^* + E_2^* + E_3 - A_1A_4\beta_2\left(b_1 - b_2\right)^2}{\left(b_1^* + b_2 + 2b_{11}\right)^2},$$
$$M_9 = \frac{E_1 + E_1^*}{16b_{11}^2\beta_2}.$$

Using equation (4), and according to the procedure to obtain $f_4(\xi,\tau)$, we can obtain $s_4(\xi,\tau)$ as

$$s_4(\xi,\tau) = N_1 e^{2\theta_2+2\theta_2^*} + N_2 e^{\theta_1+\theta_2+2\theta_2^*} + N_3 e^{2\theta_1+2\theta_2^*} + N_4 e^{2\theta_2+\theta_1^*+\theta_2^*} + N_5 e^{\theta_1+\theta_2+\theta_1^*+\theta_2^*} + N_6 e^{2\theta_1+\theta_1^*+\theta_2^*} + N_7 e^{2\theta_2+2\theta_1^*} + N_8 e^{\theta_1+\theta_2+2\theta_1^*} + N_9 e^{2\theta_1+2\theta_1^*}$$

with

$$N_1 = 2b_2E_6 - 2b_2^*E_6^* - A_2B_2,$$
$$N_3 = 2b_1E_2 - 2b_2^*E_5^* - A_3B_3,$$
$$\begin{aligned}N_2 = {} & \left(b_1 + b_2\right)E_4 - A_3B_2 - A_2B_3 - \left(b_1 + b_2^* - 2\mathrm{i}b_{22}\right) \\ & \times E_5^* + \left(b_1 - b_2^* - 2b_{21}\right)E_6^*,\end{aligned}$$
$$\begin{aligned}N_4 = {} & \left(b_2 + b_1^* + 2\mathrm{i}b_{22}\right)E_5 - B_4A_2 - A_4B_2 \\ & - \left(b_1^* + b_2^*\right)E_4^* + \left(b_2 + 2b_{21} - b_1^*\right)E_6,\end{aligned}$$
$$\begin{aligned}N_5 = {} & 2\left(b_{11} + \mathrm{i}b_{22}\right)E_3 + 2\left(b_{21} + \mathrm{i}b_{12}\right)E_4 \\ & - 2\left(b_{11} - \mathrm{i}b_{22}\right)E_3^* - 2\left(b_{21} - \mathrm{i}b_{12}\right)E_4^* \\ & - A_2B_1 - A_1B_2 - A_4B_3 - A_3B_4,\end{aligned}$$
$$\begin{aligned}N_6 = {} & \left(b_1 - b_2^* + 2b_{11}\right)E_1 + \left(b_1 + b_2^* + 2\mathrm{i}b_{12}\right)E_2 \\ & - A_3B_1 - A_1B_3 - \left(b_1^* + b_2^*\right)E_3^*,\end{aligned}$$

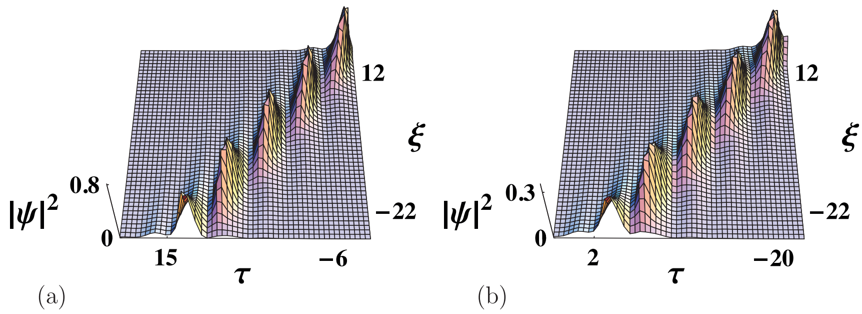


Figure 1. Breather profiles for the analytic breather solution (15) to equation (1). The parameters are $\beta_2 = -0.5$, $\tau_R = 2$, $b_{11} = 0.15$,$b_{12} = 0.5$, $b_{21} = 0.15$, $b_{22} = -1$, $k_{12} = 1.5$, $k_{22} = 0.5$, $\eta = 0.3$ with (a) $k_{11} = -2$ and $k_{21} = -1$, (b) $k_{11} = -0.5$ and $k_{21} = 1$.

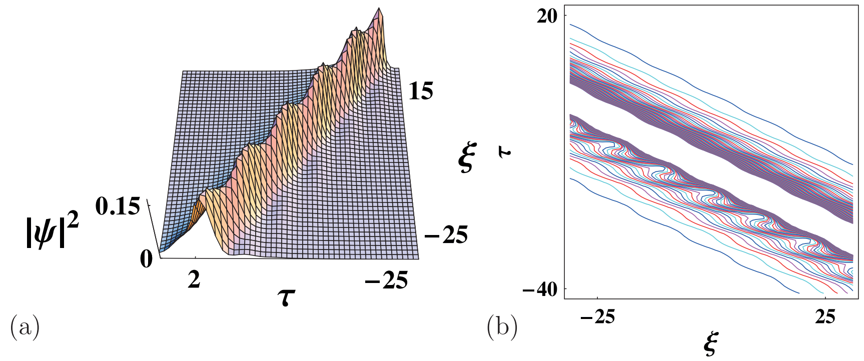


Figure 2. Breather profiles for the analytic breather solution (15) to equation (1) with the same parameters as those given in figure 1(a) but with $k_{21} = 1$.

$$
\begin{aligned}
N_7 &= 2b_2 E_5 - A_4 B_4 - 2b_1^* E_2^*,\\
N_9 &= 2b_1 E_1 - A_1 B_1 - 2b_1^* E_1^*,\\
N_8 &= \left(b_2 - b_1^* - 2b_{11}\right) E_1^* - A_4 B_1 - A_1 B_4\\
&\quad - \left(b_2 + b_1^* - 2\mathrm{i}b_{12}\right) E_2^* + (b_1 + b_2)\, E_3.
\end{aligned}
$$

Without loss of generality, we set $\varepsilon = 1$, and we can write the explicit form of analytic breather solutions as

$$\psi(\xi,\tau) = \frac{g(\xi,\tau)}{f(\xi,\tau)} = \frac{g_1(\xi,\tau) + g_3(\xi,\tau)}{1 + f_2(\xi,\tau) + f_4(\xi,\tau)}, \qquad (15)$$

where $g_1(\xi,\tau)$, $f_2(\xi,\tau)$, $g_3(\xi,\tau)$, and $f_4(\xi,\tau)$ are defined in expressions (11)–(14).

3. Discussions

By choosing the appropriate values in the analytic breather solution (15), we can present breather profiles in nondimensional form, as shown in figure 1. It is noted that we demonstrate the existence of breathers in a HC-PCF analytically for the first time. They have different periodicity properties with different values of the parameters in solution (15). Their energies are concentrated in a localized and oscillatory fashion, and the breathers oscillate in both space and time. By changing the values of k_{11} and k_{21}, we can change the amplitude, period and width of the breathers. In figure 1(b), the values of k_{11} and k_{21} are smaller than those in figure 1(a): the amplitude and period of the breathers decrease while the breather width is broadened.

The oscillating state and breather period can be controlled by choosing different parameter values. When the sign of k_{21} is positive, such as $k_{21} = 1$ in figure 2, the breather period decreases, and the oscillating state is weakened. Moreover, the amplitude of the breathers decreases. Changing the value of k_{11}, we can also control the oscillating state. In figure 3, the oscillating state of the breathers disappears when the value of k_{11} decreases, such as for $k_{11} = -20$. The pulse shape looks like the sech type, but the pulse front is steeper than a sech one.

In figures 1–3, we show the influences of k_{11} and k_{21}. k_{12} and k_{22} have an influence on the breather phase alone. Next, the influences of b_{11}, b_{12}, b_{21} and b_{22} will be discussed. By increasing the value of b_{11} or decreasing the value of b_{21} we can adjust the breathers in figure 4. The breather amplitude decreases, the breather oscillation is enhanced and the breather width is broadened. The breather amplitude oscillates with large deviations. By increasing the values of b_{12} and b_{22} we can amplify and compress the breathers as shown in figure 5. The amplitude of breathers increases gradually, and the breathers can be amplified. At the same time, the breather width becomes narrow in figure 5(b) and the breathers are compressed. So our work can be used to design laser systems for the generation of high energy pulses, which are more conducive to generating a supercontinuum.

By decreasing the value of β_2 or increasing the value of η the amplitude of the breathers can also be increased.

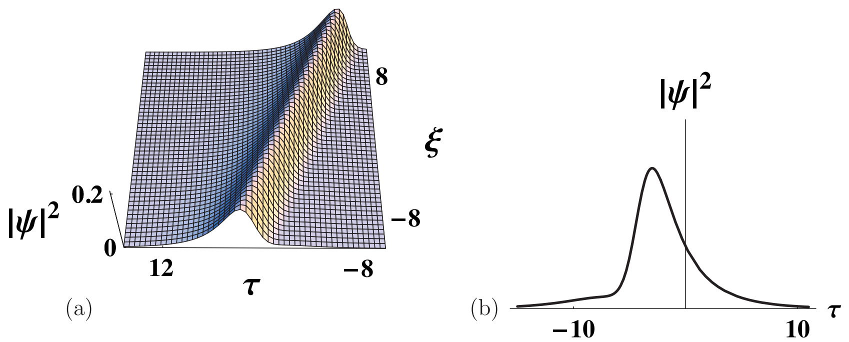


Figure 3. Breather profiles for the analytic breather solution (15) to equation (1) with the same parameters as those given in figure 1(a), but with $k_{11} = -20$.

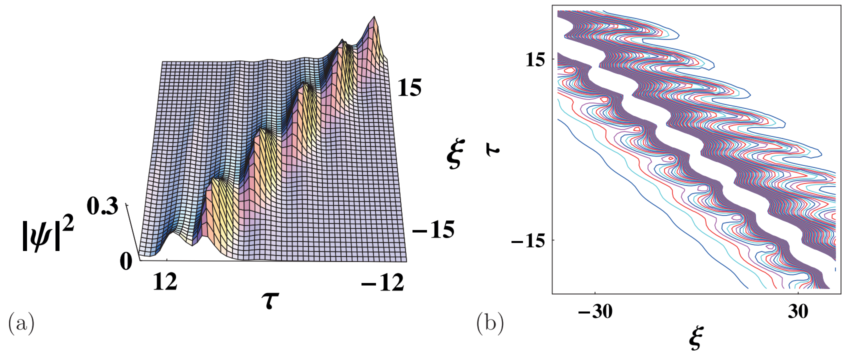


Figure 4. Breather profiles for the analytic breather solution (15) to equation (1) with the same parameters as those given in figure 1(a) but with $b_{11} = 0.3$.

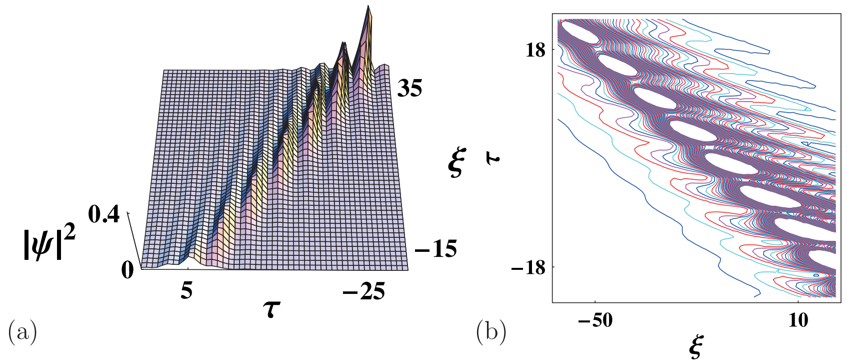


Figure 5. Breather profiles for the analytic breather solution (15) to equation (1) with the same parameters as those given in figure 1(a) but with $b_{12} = 0.6$.

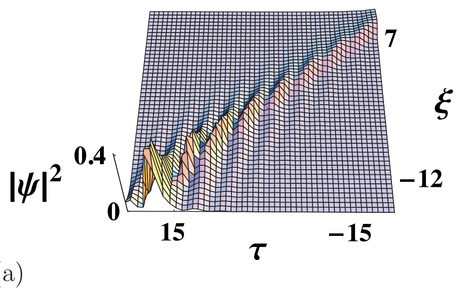
In this case, the breather profiles are similar to those in figure 4. That is, the group-velocity dispersion affects just the breather amplitude. When the Raman resonant time constant τ_R changes, such as $\tau_R = 5$ in figure 6, the breather amplitude decreases and there is an energy loss.

4. Conclusions

A class of localized nonlinear excitations, breathers, has been obtained in HC-PCFs. The NLS equation (see equation (1)), which can be used to describe the propagation of breathers in HC-PCFs, has been investigated analytically. Using the Hirota bilinear method and the auxiliary function method, an analytic breather solution (15) with eight free parameters has been presented. The characteristics of breathers have been discussed in relation to special choices of these free parameters in the solution (15). The following aspects should be noted:

(1) We demonstrate that the choice of parameters not only controls the amplitude of breathers but also influences their oscillating state and period. The amplitude, period and oscillating state of breathers can be adjusted with k_{1j},k_{2j} andb_{j1} (see figures 1–4). The group-velocity dispersion β_2 has an effect on breather amplitude.

(2) Breathers have been amplified and compressed with increasing values of b_{12} andb_{22} (see figure 5). This amplification mechanism can be used to obtain high energy pulses and generate a supercontinuum in a HC-PCF.



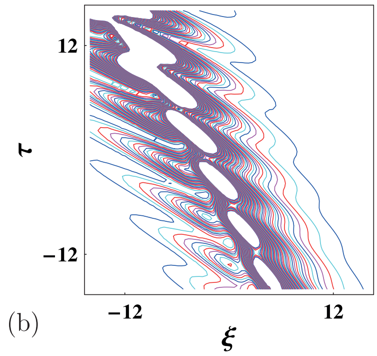


Figure 6. Breather profiles for the analytic breather solution (15) to equation (1) with the same parameters as those given in figure 1(a) but with $\tau_R = 5$.

(3) Decreasing the Raman resonant time constant τ_R can effectively prevent energy loss, as shown in figure 6.

Our results may be useful for the application of supercontinuum generation in laser systems and could also be expected to be helpful in describing pulse propagation and in relevant applications in nonlinear optics and BECs.

Acknowledgments

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References

[1] Russell P St J 2003 Photonic crystal fibers *Science* **299** 358–62
[2] Roberts P J *et al* 2005 Ultimate low loss of hollow-core photonic crystal fibres *Opt. Express* **13** 236–44
[3] Russell P St J 2006 Photonic-crystal fibers *J. Lightwave Technol.* **24** 4729–49
[4] Lebrun S, Delaye P and Roosen G 2007 Stimulated Raman scattering in hollow core photonic crystal fibres *Ann. Phys. Fr.* **32** 45–51
[5] Arriaga J and Meneses B 2003 Band structure for the cladding of a hollow core photonic crystal fibre *Rev. Mex. Fis.* **49** 335–7
[6] Cregan R F, Mangan B J, Knight J C, Birks T A, Russell P St J, Roberts P J and Allan D C 1999 Single-mode photonic band gap guidance of light in air *Science* **285** 1537–9
[7] Shinoj V K and Murukeshan V M 2011 Numerical investigation and optimisation of hollow-core photonic crystal fibre for optical trapping of fluorescent microparticles *Micro Nano Lett.* **6** 785–9
[8] Qian W W, Zhao C L, Wang Y P, Chan C C, Liu S J and Jin W 2011 Partially liquid-filled hollow-core photonic crystal fiber polarizer *Opt. Lett.* **36** 3296–8
[9] Benabid F and Roberts P J 2011 Linear and nonlinear optical properties of hollow core photonic crystal fiber *J. Mod. Opt.* **58** 87–124
[10] Roberts P J, Williams D P, Sabert H, Mangan B J, Bird D M, Birks T A, Knight J C and Russell P St J 2006 Design of low-loss and highly birefringent hollow-core photonic crystal fiber *Opt. Express* **14** 7329–41
[11] Sun Q, Mao Q H, Liu E M, Rao R Z and Ming H 2008 Hollow-core photonic crystal fiber high-pressure gas cell *Proc. SPIE* **7004** 700455
[12] Heckl O H *et al* 2009 High harmonic generation in a gas-filled hollow-core photonic crystal fiber *Appl. Phys.* B **97** 369–73
[13] Pinto A M R, Baptista J M, Santos J L, Lopez-Amo M and Frazão O 2012 Micro-displacement sensor based on a hollow-core photonic crystal fiber *Sensors* **12** 17497–503
[14] Saleh M F and Biancalana F 2011 Understanding the dynamics of photoionization-induced nonlinear effects and solitons in gas-filled hollow-core photonic crystal fibers *Phys. Rev.* A **84** 063838
[15] Saleh M F, Chang W, Hölzer P, Nazarkin A, Travers J C, Joly N Y, Russell P St J and Biancalana F 2011 Theory of photoionization-induced blueshift of ultrashort solitons in gas-filled hollow-core photonic crystal fibers *Phys. Rev. Lett.* **107** 203902
[16] Agrawal G P 2007 *Nonlinear Fiber Optics* 4th edn (San Diego, CA: Academic)
[17] Serebryannikov E E and Zheltikov A M 2007 Ionization-induced effects in the soliton dynamics of high-peak-power femtosecond pulses in hollow photonic-crystal fibers *Phys. Rev.* A **76** 013820
[18] Liu W J, Tian B, Zhen H L and Jiang Y 2012 Analytic study on solitons in gas-filled hollow-core photonic crystal fibers *Europhys. Lett.* **100** 64003
[19] Bang O and Peyrard M 1995 High order breather solutions to a discrete nonlinear Klein–Gordon model *Physica* D **81** 9–22
[20] Liu W J, Tian B, Zhang H Q, Xu T and Li H 2009 Solitary wave pulses in optical fibers with normal dispersion and higher-order effects *Phys. Rev.* A **79** 063810
[21] Liu W J, Tian B, Zhang H Q, Li L L and Xue Y S 2008 Soliton interaction in the higher-order nonlinear Schrödinger equation investigated with Hirota's bilinear method *Phys. Rev.* E **77** 066605
[22] Liu W J, Tian B and Zhang H Q 2008 Types of solutions of the variable-coefficient nonlinear Schrödinger equation with symbolic computation *Phys. Rev.* E **78** 066613
[23] Liu W J, Tian B, Xu T, Sun K and Jiang Y 2010 Bright and dark solitons in the normal dispersion regime of inhomogeneous optical fibers: soliton interaction and soliton control *Ann. Phys.* **325** 1633–43
[24] Hirota R 1971 Exact solution of the Korteweg–de Vries equation for multiple collisions of solitons *Phys. Rev. Lett.* **27** 1192–4