Analytic study on soliton amplification in graphene oxide mode-locked Er-doped fiber lasers

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Analytic study on soliton amplification in graphene oxide mode-locked Er-doped fiber lasers

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Graphene oxide mode-locked Er-doped fiber lasers are investigated in this paper. The generalized nonlinear Schrödinger equation, which can be used to describe the fiber lasers, is studied analytically. Soliton solutions for this equation are obtained, and the stable solitons are generated in the fiber lasers. Physical effects, such as the gain bandwidth, distributed gain, group-velocity dispersion, and nonlinearity of fiber lasers, are analyzed. Soliton amplification techniques are proposed which can effectively restrain the appearance of pedestals during the evolution of solitons in the cavity. These analytic results are compared with the experimental ones. The results of this paper will be valuable to the study of the fiber lasers properties and provide theoretical guidance for the obtaining of high-power ultra-short pulses.

Keywords: mode-locked Er-doped fiber lasers; graphene oxide; soliton amplification

1. Introduction

Graphene, which was discovered in 2004 [1], has been making a profound impact in the field of material science [2]. As an optical material, the graphene has been investigated as a novel saturable absorber material to mode-locked lasers [3–10], due to its strong saturable absorption and broad operational wavelength range [11]. However, since the graphene has exceptional thermal and chemical stability [12], it is insoluble in the water and organic solvents, which causes some difficulties to its preparation and application. Graphene oxide (GO), serving as a precursor for graphene, attracts much attention due to its optical properties [13,14]. As saturable absorbers, the GO becomes more flexible than the graphene due to the existence of oxygen-containing functional groups [13–17].

On the other hand, the saturable absorber is a key component for fiber lasers [9]. In saturable absorbers, high-intensity short pulses cause less loss than the low-intensity ones [9]. The semiconductor saturable absorber mirror (SESAM) is one of the most widely used saturable absorbers. However, SESAMs require complex and costly clean room-based fabrications systems and suffer from low optical damage thresholds. Single-wall carbon nanotubes (SWCNTs) and graphene have emerged as promising saturable absorbers. But SWCNTs have poor wide-band tunability, and the formation of bubbles causes high nonsaturable losses in the cavity. The graphene has significant advantages over SWCNTs including much lower threshold level of the saturable absorption, an ultra-fast recovery time, and a wide operating spectral range covering whole telecommunication bands [18]. GO has a fast energy relaxation of hot carriers and strong saturable absorption with quality comparable to that of the graphene [19,20]. Furthermore, GO is inexpensive and abundant, and could be easily fabricated by simple oxidation and ultra-sonication process [13]. Those properties make GO more suitable for the application as saturable absorbers in fiber lasers [13].

Figure 1 shows the experimental configuration of the GO mode-locked Er-doped fiber (EDF) laser [11]. An optical circulator is used to assure the unidirectional propagation of the laser and incorporate the GO saturable absorber mirror (GOSAM) into the cavity. The GOSAM was reported in Ref. [11,13,21]. The EDF was used as the gain medium with positive group velocity dispersion (GVD), pumped by a 980 nm high-power laser diode through a wavelength division multiplexer. The polarization controller (PC) was used to optimize the mode-locking operation, while a polarization independent isolator maintains the unidirectional laser pulse propagation. The GVD is one of the key factors to maintain stability of the fiber laser operation. The GO exhibited extremely large normal dispersion in comparison with SESAMs. The single-mode fiber (SMF) was added to provide enough anomalous dispersion to compensate the large normal dispersion of GO so that the net cavity dispersion became anomalous. The rest of the cavity was consisted of SMF, which has anomalous dispersion at 1560 nm. A fifty

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percent fiber coupler was used to output the mode-locked pulses.

For Figure 1, there are some experimental reports of GO mode-locked fiber lasers. Femtosecond pulse generation with the GO mode-locked fiber laser has been demonstrated [22]. A nanosecond-pulse EDF laser, which is passively mode locked by a hollow-core photonic crystal fiber filled with few-layered GO solution, has been demonstrated [23]. Taking the simpler manufacturing technology and possibility of mass production into account, GO is a good candidate as a cost-effective material for saturable absorbers for EDF lasers [21]. Besides, femtosecond Er-doped all-fiber lasers mode-locked with GO have been observed [13,24]. By employing the dispersion compensation fiber, the dissipative soliton has been obtained from the GO mode-locked EDF lasers [11,25]. Furthermore, the generation of L-band solitons from a GO mode-locked EDF laser has been demonstrated [26].

Meanwhile, numerical and analytic studies have also been done in special forms. In the presence of an arbitrary distributed gain function, the propagation of pulses in Figure 1 can be described by the generalized nonlinear Schrödinger (NLS) equation [11,27,28]:

\[
\frac{\partial A}{\partial z} = \frac{g}{2} A - i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + i \gamma |A|^2 A + \frac{g}{2 \Omega_g^2} \frac{\partial^2 A}{\partial T^2},
\]

where \( A(z, T) \) is the slowly varying pulse envelope. \( T \) is the temporal dependence, and \( z \) is longitudinal dependence along the fiber. \( g \) represents the distributed gain, \( \beta_2 \) represents the group-velocity dispersion coefficient, \( \gamma \) represents the nonlinear parameter, and \( \Omega_g \) represents the gain bandwidth.

When \( g (2\Omega_g^2)^{-1} \) is equal to zero, Equation \( (1) \) have been extensively investigated [29]. With Equation \( (1) \), several aspects concerning the numerical simulation of ultra-short pulse mode-locked fiber lasers have been highlighted [28]. They have shown that multiple attractors have been accessed by different initial conditions especially in the transient region between different mode-locking regimes. Besides, a new asymptotically exact analytic similarity solution of Equation \( (1) \) has been obtained [27]. And they have found that fiber lasers can generate a new type of linearly chirped self-similar pulses, which was called Hyper–Gaussian similarities.

However, systematic theoretical investigations to generate solitons in the GO mode-locked fiber lasers for Equation \( (1) \) have not been studied among these reports when \( g (2\Omega_g^2)^{-1} \) is not equal to zero. Here we present a theoretical study to obtain the stable solitons generated from the GO mode-locked EDF lasers for Equation \( (1) \). Soliton amplification techniques will be proposed which can effectively restrain the appearance of pedestals during the evolution of solitons in the cavity. Such physical effects as the gain bandwidth, distributed gain, group-velocity dispersion, and nonlinearity of fiber lasers will be analyzed. Moreover, these analytic results will be compared with the experimental ones in the previous literature. The obtained results will be helpful to the generation of mode-locked pulses with narrower pulse width and larger pulse energy.

Our presentation will be structured as follows. In Section 2, we present the analytic solutions for Equation \( (1) \). In Section 3, we study the features and properties of solitons, and analyze their influences on solitons and soliton amplification. Finally in Section 4, we summarize our findings and present our conclusions.

2. Analytic solutions for Equation \( (1) \)

We will use the symbolic computation and Hirota method to investigate Equation \( (1) \) analytically. By introducing the dependent variable transformation [30,31],

\[
A(z, T) = \frac{h(z, T)}{f(z, T)},
\]

where \( h(z, T) \) and \( f(z, T) \) are complex differentiable functions, we obtain the partial derivative forms of \( A(z, T) \) as

\[
\frac{\partial A}{\partial z} = \frac{D_z h \cdot f}{f^2}, \quad \frac{\partial^2 A}{\partial T^2} = \frac{D_T^2 h \cdot f}{f^2} - h \frac{D_T^2 f \cdot f}{f^2}.
\]

Here, Hirota’s bilinear operators \( D_z \) and \( D_T \) [31] are defined by

\[
D_z^m D_T^n (a \cdot b) = \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z'} \right)^m \left( \frac{\partial}{\partial T} - \frac{\partial}{\partial T'} \right)^n a(z, T) b(z', T') \bigg|_{z' = z, T' = T}.
\]
Straightforward substitution of Expression (3) into Equation (1) gives rise to the following equation:

$$\frac{D_z h \cdot f}{f^2} = \frac{g}{2h} + \left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) \left( \frac{D_T^2 h \cdot f}{f^2} - h \frac{D_T^2 f \cdot f}{f^2} \right) + i\gamma h^2 h^*, \tag{5}$$

where $*$ denotes the complex conjugate.

Equation (5) can be rewritten as:

$$\left[ D_z h \cdot f - \frac{g}{2h} h \cdot f - \left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) D_T^2 h \cdot f \right] \frac{1}{f^2} = \left[ i\gamma h^2 h^* - \left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) D_T^2 f \cdot f \right] \frac{h}{f^3}. \tag{6}$$

Thus, the bilinear forms for Equation (1) can be obtained as:

$$D_z h \cdot f - \frac{g}{2h} h \cdot f - \left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) D_T^2 h \cdot f = 0,$$  \tag{7}

$$\left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) D_T^2 f \cdot f = i\gamma h^2 h^*. \tag{8}$$

Equations (7) and (8) can be solved by the following power series expansions for $h(z, T)$ and $f(z, T)$:

$$h(z, T) = \varepsilon h_1(z, T), \quad f(z, T) = 1 + \varepsilon^2 f_2(z, T), \tag{9}$$

where $\varepsilon$ is a formal expansion parameter. Substituting Expression (9) into Equations (7) and (8), and collecting the coefficients of the same powers of $\varepsilon$, we get

$$\varepsilon : B_1(h_1 \cdot 1) = 0, \tag{10}$$

$$\varepsilon^2 : B_2(f_2 \cdot 1 + 1 \cdot f_2) = i\gamma h_1 h_1^*, \tag{11}$$

$$\varepsilon^3 : B_1(h_1 \cdot f_2 + h_3 \cdot 1) = 0, \tag{12}$$

where $B_1 = \left[ D_z - \frac{g}{2} - \left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) D_T^2 \right]$ and $B_2 = \left( \frac{g}{2\Omega^2_g} - i \frac{\beta_g}{2} \right) D_T^2$.

To obtain the one-soliton solutions for Equation (1), we assume that

$$h_1(z, T) = \exp \left[ (a_{11} + i a_{12}) z + (b_{11} + i b_{12}) T + k_{11} + i k_{12} \right], \tag{13}$$

where $a_{1j}$’s, $b_{1j}$’s, and $k_{1j}$’s ($j = 1, 2$) are the real constants. Substituting $h_1(z, T)$ into Equation (10), we get the constraints on the parameters:

$$a_{11} = \left( b_{11}^2 - b_{12}^2 \right) \frac{g}{2\Omega^2_g} + \frac{g}{2} + b_{11} b_{12} \beta_g,$$

$$a_{12} = \frac{g b_{11} b_{12}}{\Omega^2_g} - \frac{b_{11}^2 \beta_g}{2} + \frac{b_{12}^2 \beta_g}{2}. \tag{14}$$

Substituting Expression (13) into Equation (11), we derive

$$f_2(z, T) = -\frac{\gamma}{2} \frac{2b_{11}^2 \beta_g^2 + 4g}{4b_{11}^2 \beta_g^2 + 4g} \times e^{i\frac{g}{2\Omega^2_g} b_{11} b_{12} \beta_g z + i\frac{b_{11}^2 + b_{12}^2}{2\Omega_g^2}} \left[ 2b_{11} + 2b_{11} \sqrt{b_{11}^2 + b_{12}^2} \right]. \tag{14}$$

And then substituting Expressions (13) and (14) into Equation (12) yields $b_{12} = 0$, $|b_{11}| = \Omega_g$. 

Figure 2. (a) Propagation of the stable soliton. Parameters are: $\Omega_g = 0.5$, $\beta_g = -10$ ps$^2$m$^{-1}$, $k_{11} = 1.5$, $k_{12} = 5$, $g = 1$ dB, and $\gamma = 1$ W$^{-1}$m$^{-1}$; (b) Soliton compression at $z = 3$ between $\Omega_g = 0.5$ (black solid line) and $\Omega_g = 2$ (red dash-dotted line). (The colour version of this figure is included in the online version of the journal.)

Figure 3. Soliton amplification in the GO mode-locked EDF laser with different GVD. (The colour version of this figure is included in the online version of the journal.)
Without loss of generality, we set $\varepsilon = 1$, and the one-soliton solutions can be expressed as:

$$A(z, T) = \frac{h_1(z, T)}{1 + f_2(z, T)} e^{(a_{11} + a_{12}) z + (b_{11} + b_{12}) T + c_{11} + c_{12}}$$

$$= \frac{\gamma \Omega_2}{1 - \frac{\gamma \Omega_2}{4b_{11}(\beta_2 \Omega_2^2 + i\varepsilon)}} e^{g z + 2b_{11} \beta_2 g z + \frac{g(\varepsilon_1^2 - \varepsilon_2^2) \varepsilon}{4b_{11}^2} + 2b_{11} T + 2c_{11}}$$

(15)

### 3. Discussion

With appropriate parameters in Solution (15), the stable soliton is obtained as shown in Figure 2(a). The amplitude and width of the soliton remain unchanged from the output of the GO mode-locked EDF laser. The properties of the solitons are also maintained. To verify that the mode locking is resulted from the GO, a broadband mirror has been used to replace the GOSAM from the cavity, and then no mode locking has been observed [27]. In fact, the GO has a fast energy relaxation of hot carriers and strong saturable absorption, and the saturable absorption in the GO is attributed to the presence of pristine graphene nanoislands with $sp^2$-hybridized carbon atoms [23]. Increasing the value of gain bandwidth $\Omega_g$, we can amplify the soliton and increase the peak power of the solitons in Figure 2(b). Because the gain bandwidth increases, the new frequency component resulted from the enhancement of the self-phase modulation effect is effectively enlarged, and the peak power is increased. Different from the reported results in Ref. [11], the amplified soliton is pedestal free. When the value of $\Omega_g$ increases to a certain degree, the amplitude $|A(z, T)|^2$ increases. Thus, a small value of $\Omega_g$ is preferable for stable single-frequency operation.

When the absolute value of group-velocity dispersion coefficient $\beta_2$ increases, the soliton is amplified in Figure 3. The pulse width is not changed and the soliton amplitude increases. Moreover, the amplified solitons are without pedestals. In Figure 3, the black solid line represents the soliton profile with $\beta_2 = -10$ at $z = 3$. The red dashed line is the soliton profile with $\beta_2 = -20$. The soliton is amplified with the increasing value of $\beta_2$. For Figure 4, we can change the length of the SMF to increase $\beta_2$. Because the GO has extremely large normal dispersion in comparison with SESAMs, we should increase large negative dispersion to carry out the soliton amplification. This result is in agreement with the experimental one [13]. The intra-cavity amplification technique helps improve the signal-to-noise ratio of the GO mode-locked EDF laser.

Increasing the distributed gain $g$ or decreasing the nonlinear parameter $\gamma$, we can also amplify the soliton as shown in Figure 4. The black solid line denotes the soliton profile with $g = 1$, and the red dash-dotted line denotes the soliton profile with $g = 4$ in Figure 4(a). That means the higher the doping concentration of the gain medium, the greater the soliton energy in the same conditions will be. This result is also consistent with the experimental one. In Figure 4(b), the black solid line denotes the soliton profile with $\gamma = 1$, and the red dash-dotted line denotes the soliton profile with $\gamma = 0.5$. It indicates that decreasing the nonlinear effects of materials in the laser cavities will amplify the soliton. Of course, the nonlinear effects cannot be too small. The value should be maintained in a certain range to support the solitons. Hence, soliton amplification can be realized by changing the values of $g$ and $\gamma$.

### 4. Conclusions

In conclusion, the generalized NLS equation [see Equation (1)], which can be used to describe the propagation of pulses in the GO mode-locked EDF lasers, has been investigated analytically. The stable solitons have been obtained in the GO mode-locked EDF lasers. The soliton amplification techniques have been advised by changing the gain bandwidth, group-velocity dispersion, distributed gain, and nonlinearity of fiber lasers (see Figures 2–4). The amplified solitons have been without pedestals. Our studies have shown that the GO could be a promising saturable absorber to achieve mode locking and realize soliton amplification without pedestals. Corresponding experimental results will be reported in future.
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